**INSTRUCTIONS**

Homework solutions should be neatly written or typed and submitted through Gradescope. No work can be accepted outside of this system, and no late work will be accepted. Please ensure that your submission is legible (neatly written and not too faint) or your homework may not be graded. You may update your submission as many times as you’d like up to the deadline. Only the most recent submission will be graded.

Thirty problems from homework assignments will be graded randomly throughout the quarter. You will not know in advance which problems, if any, will be graded on each assignment.

You may consult your textbook, class notes, lecture slides, instructors, TAs, and tutors for help with homework. You may also discuss homework questions with classmates, but you may not share written work with classmates. You must write up your solutions alone, in your own words. The assignments have been developed to facilitate your learning and to provide a method for fairly evaluating your knowledge and abilities, not the knowledge and abilities of others. To facilitate learning, you are authorized to discuss assignments with others; however, to ensure fair evaluations, you are not authorized to view or share written work with another person, or to write your submission in collaboration with another person. You should not look for answers to homework problems in other texts or sources, including the internet.

Do not post about homework questions on Piazza. For help with homework, please consult the course textbook, lecture slides, class notes, and podcasts, or come visit us in office hours.

**READING:** Highlights of Chapter 7
1. It is not currently known whether NP is closed under complementation.

(a) Suppose it were shown that NP was closed under complementation. Would that imply that \( P = NP \), \( P \neq NP \), or would it give us no information about \( P \) versus \( NP \)? Justify your answer.

(b) Suppose it were shown that NP was not closed under complementation. Would that imply that \( P = NP \), \( P \neq NP \), or would it give us no information about \( P \) versus \( NP \)? Justify your answer.

2. In this problem, we will show that a new language, which we will call \( SEPARATE \), is NP-complete by showing it is polynomial time reducible from \( CLIQUE \). Let

\[
SEPARATE = \{ \langle G, k \rangle \mid G \text{ is an undirected graph with a set } S \text{ of } k \text{ vertices,}
\]
\[
\text{none of which have an edge to any other vertex in } S \}
\]

Prove that you can decide \( CLIQUE \) if and only if you can decide \( SEPARATE \). That is, show that you have a procedure to answer the question of whether an arbitrary graph has a \( k \)-clique if and only if you have a procedure to answer the question of whether an arbitrary graph has a set \( S \) of \( k \) vertices, none of which have an edge to any other vertex in \( S \).

Illustrate your proof with an example graph.