INSTRUCTIONS

Homework solutions should be neatly written or typed and submitted through Gradescope. No work can be accepted outside of this system, and no late work will be accepted. Please ensure that your submission is legible (neatly written and not too faint) or your homework may not be graded. You may update your submission as many times as you’d like up to the deadline. Only the most recent submission will be graded.

Thirty problems from homework assignments will be graded randomly throughout the quarter. You will not know in advance which problems, if any, will be graded on each assignment.

You may consult your textbook, class notes, lecture slides, instructors, TAs, and tutors for help with homework. You may also discuss homework questions with classmates, but you may not share written work with classmates. You must write up your solutions alone, in your own words. The assignments have been developed to facilitate your learning and to provide a method for fairly evaluating your knowledge and abilities, not the knowledge and abilities of others. To facilitate learning, you are authorized to discuss assignments with others; however, to ensure fair evaluations, you are not authorized to view or share written work with another person, or to write your submission in collaboration with another person. You should not look for answers to homework problems in other texts or sources, including the internet.

Do not post about homework questions on Piazza. For help with homework, please consult the course textbook, lecture slides, class notes, and podcasts, or come visit us in office hours.

READING: Sipser 4.1, 4.2
Keep in mind that you can use without proof any theorems or constructions proven in class or in your textbook. Please cite the relevant theorem or construction when you want to use it, either by briefly summarizing it or calling it by name or theorem number.

Section 4.1, the lecture slides and discussion worksheet from February 24, and the solutions to selected Chapter 4 exercises provide many examples of how to show a language is decidable.

When proving a language is decidable, give both a construction of a Turing machine that decides the language, and a proof that your construction is correct (see lecture slides).

1. Show that the following language is decidable.

   \[ L_{00} = \{ \langle R \rangle \mid R \text{ is a regular expression over } \{0, 1\} \text{ that generates some string with two consecutive zeros} \} \]

2. For each part, give an example of sets \( A \) and \( B \) satisfying the conditions, or explain why it is impossible. A set is called countably infinite if it is both countable and infinite.

   (a) \( A \) is a proper subset of \( B \), \( A \) is countably infinite, \( B \) is countably infinite
   (b) \( A \) is a proper subset of \( B \), \( A \) is uncountable, \( B \) is countably infinite
   (c) \( A \) is a proper subset of \( B \), \( A \) is countable, \( B \) is uncountable
   (d) \( A \) is a proper subset of \( B \), \( A \) is countably infinite, \( B \) is uncountable
   (e) \( A \) is a proper subset of \( B \), \( A \) is uncountable, \( B \) is uncountable