INSTRUCTIONS

Homework should be done in groups of one to three people. You are free to change group members at any time throughout the quarter. Problems should be solved together, not divided up between partners. A single representative of your group should submit your work through Gradescope. Submissions must be received by 11:59pm on the due date, and there are no exceptions to this rule.

Homework solutions should be neatly written or typed and turned in through Gradescope by 11:59pm on the due date. No late homeworks will be accepted for any reason. You will be able to look at your scanned work before submitting it. Please ensure that your submission is legible (neatly written and not too faint) or your homework may not be graded.

Students should consult their textbook, class notes, lecture slides, instructors, TAs, and tutors when they need help with homework. Students should not look for answers to homework problems in other texts or sources, including the internet. Only post about graded homework questions on Piazza if you suspect a typo in the assignment, or if you don’t understand what the question is asking you to do. Other questions are best addressed in office hours.

Your assignments in this class will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should always explain how you arrived at your conclusions, using mathematically sound reasoning. Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to convince the reader that your results and methods are sound.

For questions that require pseudocode, you can follow the same format as the textbook, or you can write pseudocode in your own style, as long as you specify what your notation means. For example, are you using “=” to mean assignment or to check equality? You are welcome to use any algorithm from class as a subroutine in your pseudocode. For example, if you want to sort list A using InsertionSort, you can call InsertionSort(A) instead of writing out the pseudocode for InsertionSort.

REQUIRED READING Rosen 10.1, 10.2, 10.3, 10.4 through Theorem 1, 10.5 through Example 7.

KEY CONCEPTS Graphs (definitions, modeling problems using graphs), Hamiltonian tours, Eulerian tours, Fleury’s algorithm, DAGs.
1. (a) (3 points) Which of the edges in the graph above are bridges?

(b) (3 points) Use Fleury’s algorithm to find an Eulerian tour of the graph above. Suppose that whenever the algorithm allows you a choice for which edge to take, you always take the edge whose label comes first alphabetically. For example, if you were in a position where you could take edge b, edge c, or edge h, you would take edge b. Write down the Eulerian tour you find by listing the edges of your tour in order.

(c) (3 points) Draw a connected graph with 5 vertices that has no Eulerian tour.

2. We say a matrix has dimensions $m \times n$ if it has $m$ rows and $n$ columns. If matrix A has dimensions $x \times y$ and matrix B has dimensions $z \times w$, then the product $AB$ exists if and only if $y = z$. In the case where the product exists, $AB$ will have dimensions $x \times w$. In this problem, we are given a list of matrices and their dimensions, and we want to determine if there is an order in which we can multiply all the matrices together, using each matrix exactly once. For example, here is a possible list of matrices and their dimensions:

- A is $3 \times 5$
- B is $4 \times 3$
- C is $4 \times 4$
- D is $2 \times 5$
- E is $5 \times 2$
- F is $5 \times 3$

(a) (3 points) Given any list of matrices and dimensions, describe how to draw a graph so that each order in which we can multiply the matrices corresponds to a Hamiltonian tour of your graph. Carefully describe what the vertices of your graph represent, and when two vertices are connected with an edge.

(b) (1 point) Draw the graph described in part (a) for the example list of matrices given above.

(c) (3 points) Given any list of matrices and dimensions, describe how to draw a graph so that each order in which we can multiply the matrices corresponds to an Eulerian tour of your graph. Carefully describe what the vertices of your graph represent, and when two vertices are connected with an edge.

(d) (1 point) Draw the graph described in part (c) for the example list of matrices given above.

(e) (2 points) For the given example list of matrices, give one order in which we can multiply those matrices, or say that no such order exists.
3. Draw a graph representing the streets on which a mail carrier will walk in order to deliver mail to each house on each side of the 12 rectangular blocks in the figure above. Assume that by walking down the street once, the carrier can deliver mail to houses on both sides of the street. You do not need to turn in your graph but it should help you to answer the following questions.

(a) (2 points) If the mail carrier starts at the corner of Ivy and Front Street, is it possible for the mail carrier to deliver mail to each house and business without walking down a street twice? What is such a route called using the language of graph theory?

(b) (2 points) In this mail zone, the carrier delivers to each side of the 12 blocks, arranged in a 3 by 4 grid. If another mail zone consists of \( mn \) blocks, arranged in an \( m \) by \( n \) grid, and again the carrier must deliver to each side of the \( mn \) blocks, how many vertices and edges will the associated graph have?

(c) (2 points) In the general \( m \) by \( n \) mail zone described in part (b), how many vertices of degree 2 are there? Degree 3? Degree 4?

(d) (2 points) In the general \( m \) by \( n \) mail zone described in part (b), what is the sum of the degrees of all vertices in the graph? How does this compare to the number of edges in the graph?

(e) (2 points) In the general \( m \) by \( n \) mail zone described in part (b), for what values of \( m \) and \( n \) will the carrier be able to deliver mail to each house and business without walking down a street twice? He can start anywhere in the mail zone.

4. You are planning on taking a road trip, and you have a list of cities that you might want to visit on this trip: city 1, city 2, \ldots\, city \( n \). You make an undirected graph where the vertices are cities on your list, and you connect two cities with an edge if you are willing to drive between them in one day. Let \( A \) be the adjacency matrix for this graph, and let \( a_{ij} \) represent the entry in row \( i \) and column \( j \) of \( A \). Let \( A^k \) be the adjacency matrix \( A \) raised to the \( k \)th power, and let \( a_{ij}^{(k)} \) represent the entry in row \( i \) and column \( j \) of \( A^k \).
(a) (1 point) What does it mean in terms of your road trip if $a_{21} = 0$?

(b) (2 points) What does it mean in terms of your road trip if $a_{23}a_{31} = 0$?

(c) (2 points) What does it mean in terms of your road trip if $a_{21}^{(2)} = 0$?

(d) (2 points) What does it mean in terms of your road trip if $a_{ij}^{(k)} = 0$?

(e) (2 points) For $r$ a nonnegative integer, what does it mean in terms of your road trip if $a_{ij}^{(k)} = r$?

(f) (2 points) For $r$ a nonnegative integer, what does it mean in terms of your road trip if $a_{ij} + a_{ij}^{(2)} + \cdots + a_{ij}^{(k-1)} + a_{ij}^{(k)} = r$?

5. A daily flight schedule is a list of all the flights taking place that day. In a daily flight schedule, each flight $F_i$ has an origin airport $OA_i$, a destination airport $DA_i$, a departure time $d_i$ and an arrival time $a_i > d_i$. This is an example of a daily flight schedule for February 10, 2016, listing flights as $F_i = (OA_i, DA_i, d_i, a_i)$:

- $F_1 = (BWI, MIA, 6:00am, 8:00am)$
- $F_2 = (BWI, JFK, 8:00am, 9:00am)$
- $F_3 = (MIA, ATL, 8:30am, 10:00am)$
- $F_4 = (JFK, ATL, 10:00am, 12:30pm)$
- $F_5 = (ATL, BWI, 12:00pm, 2:00pm)$
- $F_6 = (MIA, JFK, 2:30pm, 5:00pm)$

(a) (4 points) Describe how to construct a DAG so that paths in the DAG represent possible sequences of connecting flights a person could take. What are the vertices, and when are two vertices connected with an edge?

(b) (2 points) Why is your graph always a DAG?

(c) (2 points) Draw the DAG you described for the given example of February 10, 2016.

(d) (2 points) Use your DAG to help you determine the maximum number of connecting flights a person could take on February 10, 2016.