CSE 200 Final Exam
Due Wed. March 16 at 11:59 PM (OK, I won’t actually check until I get in in
Thursday morning. But all exams must be turned in before I arrive in my
office.)

Answer all parts of all questions with informal, but complete, proofs. You may
not discuss this exam with anyone except myself, whether taking the course or
not. Weights are given in the question. You may cite without proof any result
from the Arora-Barak text or proved in class. In particular, you can use with-
out proof the NP-completeness of any problem proved NP-complete in class,
in the two texts, or on the homeworks, including: SAT, 3-SAT, Independent
Set, Clique, Vertex Covering, 3-coloring, Hamiltonian Circuit, and Subset Sum.
However, you may not use the list of NP-complete problems in the appendix of
Garey and Johnson without proof.

**NP-completeness: 10 points** Show that the following problem is NP-complete:

**Problem:** All distinct triples (ADT).

**Instance:** A set of \( n \) elements \( V \), and a set of triples \( S_1, \ldots, S_m \), where each \( S_i \subset V \) and \( |S_i| = 3 \) (i.e., the three elements of \( S_i \) are all distinct).

**Solution format:** A coloring of the elements \( V \) assigning each one of
three colors, \( R, G \) and \( B \)

**Constraints:** For any triple \( S_i \), there must be exactly one vertex in \( S_i \)
of each color.

**Objective:** Decide whether there is a three coloring of \( V \) meeting the
constraints, i.e., where all triples have distinct colors.

Note: Set systems over a common universe are sometimes called hyper-
graphs. In this notation, \( V \) would be called the set of vertices, and the
\( S_i \)'s would be hyperedges of order 3. Sorry for mixing notation in the
original version.

**Closure properties of classes of problems: 5 points each class** If \( L \) is a
decision problem, let \( ALT_L \) be the problem of, given a set \( x_1, \ldots, x_n \) of
instances to \( L \) and an integer \( 0 \leq k \leq n \), deciding whether at least \( k \)
of the \( x_i \) are in \( L \). (ALT stands for “At Least a Threshold”). Say that a
class \( C \) is closed under ALT if, for any \( L \), if \( L \in C \), then \( ALT_L \in C \).

Show that the following classes are closed under ALT

1. \( RE \), the class of recursively enumerable (or computably recognizable)
   languages.
2. \( NP \)
3. \( NL \), the class of problems solvable in non-deterministic log space
4. **BPP**, the class of problems solvable in probabilistic polynomial time

**Closure properties of classes: 5 points each class** If \( L \) is a decision problem, let \( AMT_L \) be the problem of, given a set \( x_1, \ldots, x_n \) of instances to \( L \) and an integer \( 0 \leq k \leq n \), deciding whether at most \( k \) of the \( x_i \) are in \( L \). (AMT stands for “At Most a Threshold”). Say that a class \( C \) is closed under \( AMT \) if, for any \( L \), if \( L \in C \), then \( AMT_L \in C \).

For each of the classes from the previous problem, say whether they are known to be closed under \( AMT \), known not to be closed under \( AMT \), or whether it is an open problem, and explain your answer. (Hint: what other closure property does a class need to have to be closed under \( AMT \)?)