CSE 20
DISCRETE MATH

WINTER 2016

http://cseweb.ucsd.edu/classes/wi16/cse20-ab/

Today's learning goals

- Translate sentences from English to propositional logic using appropriate propositional variables and boolean operators.
- Truth tables: negation, conjunction, disjunction, exclusive or, conditional, biconditional operators.
- Evaluate the truth value of a compound proposition given truth values of its constituent variables.
- Form the converse, contrapositive, and inverse of a given conditional statement.

About you

Remote frequency: CA

To change your remote frequency
1. Press and hold power button until flashing
2. Enter two-letter code
3. Checkmark / green light indicates success

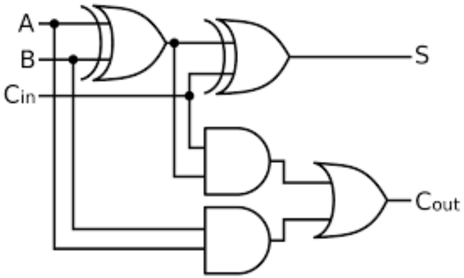
How many people in this class have you met so far?

A. None.
B. Less than 5.
C. 5-10.
D. 10-15.
E. More than 15.

Logic

Rosen Section 1.1

- Use gates and circuits to express arithmetic.



- Precisely express theorems and invariant statements.

- Make valid arguments to prove theorems.

Definitions

Rosen p. 2-4

- **Proposition**: declarative sentence that is T or F (not both)
- **Propositional variable**: variables that represent propositions.
- **Compound proposition**: new propositions formed from existing propositions using logical operators.
- **Truth table**: table with 1 row for each of the possible combinations of truth values of the input and an additional column that shows the truth value of the result of the operation corresponding to a particular row.

Propositions

Which of the following is a proposition?

A. Answer this question.
B. What time is it?
C. $4 + x = 5$.
D. $2^3 > 8$.
E. None of the above.

Compound propositions

Rosen p. 3-4

p	¬ p
T	F
F	T

p	q	p ∨ q p OR q	p ∧ q p AND q	p ⊕ q p XOR q
T	T	T	T	F
T	F	T	F	T
F	T	T	F	T
F	F	F	F	F

"p OR q is T if at least one of p or q is T"

"p AND q is T if both p and q are T"

"p XOR q is T if exactly one of p and q is T"

Compound propositions

Rosen p. 3-4

p	$\neg$ p
T	F
F	T

Negation

p	q	p v q p OR q	p ∧ q p AND q	p ⊕ q p XOR q
T	T	T	T	F
T	F	T	F	T
F	T	T	F	T
F	F	F	F	F

Disjunction Conjunction

Compound propositions

Rosen p. 10

Consider the compound proposition

$$\neg(\neg p \vee \neg q)$$

How many rows are in its truth table?

A. 1
B. 2
C. 4
D. 8
E. None of the above.

Compound propositions

Rosen p. 10

Consider the compound proposition

$$\neg(\neg p \lor \neg q)$$

p	q	$\neg(\neg p \lor \neg q)$
T	T	?
T	F	?
F	T	?
F	F	?

Compound propositions

Rosen p. 10

Consider the compound proposition

$$\neg(\neg p \vee \neg q)$$

p	q	$\neg(\neg p \vee \neg q)$
T	T	?
T	F	?
F	T	?
F	F	?

What's the value of

$$\neg(\neg p \vee \neg q)$$

when p is T and q is F?

A. T
B. F

Compound propositions

Rosen p. 10

Consider the compound proposition

$$\neg(\neg p \vee \neg q)$$

p	q	$\neg(\neg p \vee \neg q)$
T	T	?
T	F	F
F	T	?
F	F	?

To fill in rows

Plug in values one row at a time.

OR

Use intermediate columns.

Compound propositions

Rosen p. 10

Consider the compound proposition

$$\neg(\neg p \lor \neg q)$$

p	q	$\neg p$	$\neg q$	$\neg p \lor \neg q$	$\neg(\neg p \lor \neg q)$
T	T				?
T	F				F
F	T				?
F	F				?

Compound propositions

Rosen p. 10

Consider the compound proposition

$$\neg(\neg p \vee \neg q)$$

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$\neg(\neg p \vee \neg q)$
T	T	**F**			?
T	F	**F**			F
F	T	**T**			?
F	F	**T**			?

Compound propositions

Rosen p. 10

Consider the compound proposition

$$\neg(\neg p \vee \neg q)$$

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$\neg(\neg p \vee \neg q)$
T	T	F	**F**		?
T	F	F	**T**		F
F	T	T	**F**		?
F	F	T	**T**		?

Compound propositions

Rosen p. 10

Consider the compound proposition

$$\neg(\neg p \vee \neg q)$$

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$\neg(\neg p \vee \neg q)$
T	T	F	F	**F**	?
T	F	F	T	**T**	F
F	T	T	F	**T**	?
F	F	T	T	**T**	?

Compound propositions

Rosen p. 10

Consider the compound proposition

$$\neg(\neg p \vee \neg q)$$

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$\neg(\neg p \vee \neg q)$
T	T	F	F	F	**T**
T	F	F	T	T	**F**
F	T	T	F	T	**F**
F	F	T	T	T	**F**

Does this look familiar?

Logical equivalences

Rosen p. 25

Compound propositions that have the same truth values in all possible cases are **logically equivalent**, denoted $\equiv$.

p	**q**	$\neg(\neg p \vee \neg q)$
T	T	T
T	F	F
F	T	F
F	F	F

What compound proposition is logically equivalent to $\neg(\neg p \vee \neg q)$?

A. $p \wedge q$
B. $p \vee q$
C. $p \wedge \neg p$
D. $q \vee \neg q$
E. None of the above.

Translation

Rosen p. 22: 1.2#7

Express the sentence
"The message was sent from an unknown system but it was not scanned for viruses" using the propositions

p: "The message is scanned for viruses"
q: "The message was sent from an unknown system"

A. $p \wedge q$
B. $p \wedge \neg q$
C. $\neg p \vee q$
D. $p \vee \neg q$
E. None of the above.

Conditionals

Rosen p. 6-10

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

"If p, then q"

Conditionals

Rosen p. 6-10

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

"If p, then q"

The only way to make a conditional statement false is to …

Conditionals

Rosen p. 6-10

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

Hypothesis
Antecedent

Conclusion
Consequent

"If p, then q"

Conditionals

Rosen p. 6-10

Which of these compound propositions **is not** logically equivalent to p → q ?

A. $\neg p \vee q$

B. $\neg(p \wedge \neg q)$

C. $q \rightarrow p$

D. $\neg q \rightarrow \neg p$

E. None of the above.

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

Conditionals

Rosen p. 6-10

p	q	p → q	q → p	$\neg q \rightarrow \neg p$	$\neg p \rightarrow \neg q$
T	T	T			
T	F	F			
F	T	T			
F	F	T			

Converse of p→q

Contrapositive of p→ q

Inverse of p→q

Conditionals

Rosen p. 6-10

p	q	p → q	q → p	$\neg q \rightarrow \neg p$	$\neg p \rightarrow \neg q$
T	T	T			
T	F	F		F	
F	T	T	F		F
F	F	T			

Converse of p→q

Contrapositive of p→ q

Inverse of p→q

Conditionals

Rosen p. 6-10

p	q	p → q	q → p	$\neg q \rightarrow \neg p$	$\neg p \rightarrow \neg q$
T	T	T	T	T	T
T	F	F	T	F	T
F	T	T	F	T	F
F	F	T	T	T	T

Converse of p→q

Contrapositive of p→ q

Inverse of p→q

Biconditionals

Rosen p. 6-10

Which of these compound propositions **is** logically equivalent to $p \leftrightarrow q$?

A. $p \to q$

B. $p \wedge q$

C. $p \vee q$

D. $p \oplus q$

E. None of the above.

"If and only if"
"Necessary and sufficient"

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Translation

Rosen p. 22: 1.2#7

Express the sentence
"The message is scanned for viruses whenever the message was sent from an unknown system" using the propositions

p: "The message is scanned for viruses"
q: "The message was sent from an unknown system"

A. $p \wedge q$
B. $p \vee q$
C. $p \rightarrow q$
D. $p \leftrightarrow q$
E. None of the above.

Translation

Rosen p. 22: 1.2#7

Express the sentence
"It is necessary to scan the message for viruses whenever it was sent from an unknown system" using the propositions

p: "The message is scanned for viruses"
q: "The message was sent from an unknown system"

A. $p \wedge q$
B. $p \vee q$
C. $p \rightarrow q$
D. $p \leftrightarrow q$
E. None of the above.

Translation

Rosen p. 22: 1.2#7

Express the sentence
"It is necessary to scan the message for viruses whenever it was sent from an unknown system" using the propositions

p: "The message is scanned for viruses"
q: "The message was sent from an unknown system"

A. $p \wedge q$
B. $p \vee q$
C. $p \rightarrow q$
D. $p \leftrightarrow q$
E. None of the above.

Underlying logical structure of statements.

Circuits

similar to Rosen p. 24 #42

- Construct a combinatorial circuit using inverters, OR gates, and AND gates that produces the output

$$\neg p \vee (\neg q \vee \neg r)$$

Circuits

similar to Rosen p. 24 #42

- Construct a combinatorial circuit using inverters, OR gates, and AND gates that produces the output

$$\neg(p \wedge (q \wedge r))$$

Circuits

similar to Rosen p. 24 #42

- Do these two circuits always have the same output?

$$\neg p \vee (\neg q \vee \neg r) \qquad\qquad \neg(p \wedge (q \wedge r))$$

Circuits

similar to Rosen p. 24 #42

- Do these two circuits always have the same output?

$$\neg p \vee (\neg q \vee \neg r) \qquad\qquad \neg(p \wedge (q \wedge r))$$

- The same as $\neg p \vee (r \rightarrow \neg q)$?

Reminders

- Homework 2 due tomorrow
  - Integer representations
  - Algorithms
- Office hours
- No discussion on Monday; recommended practice questions on website instead