INSTRUCTIONS

Homework should be done in groups of one to three people. You are free to change group members at any time throughout the quarter. Problems should be solved together, not divided up between partners. A single representative of your group should submit your work through Gradescope. Submissions must be received by 11:59pm on the due date, and there are no exceptions to this rule.

Homework solutions should be neatly written or typed and turned in through Gradescope by 11:59pm on the due date. No late homeworks will be accepted for any reason. You will be able to look at your scanned work before submitting it. Please ensure that your submission is legible (neatly written and not too faint) or your homework may not be graded.

You may consult your textbook, class notes, lecture slides, instructors, TAs, and tutors for help with homework. You should not look for answers to homework problems in other texts or sources, including the internet. Only post about graded homework questions on Piazza if you suspect a typo in the assignment, or if you don’t understand what the question is asking you to do. Other questions are best addressed in office hours.

Your assignments in this class will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should always explain how you arrived at your conclusions, using mathematically sound reasoning. Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to convince the reader that your results and methods are sound.

For questions that require pseudocode, you can follow the same format as the textbook, or you can write pseudocode in your own style, as long as you specify what your notation means. For example, are you using “=” to mean assignment or to check equality? You are welcome to use any algorithm from class as a subroutine in your pseudocode.

READING Rosen Sections 1.4, 1.5, 1.6, 1.7, 1.8.

KEY CONCEPTS Predicates, domain of discourse / universe, existential quantifier, universal quantifier, restricting the domain, negated quantifiers, nested quantifiers, proof strategies, counterexample, (constructive) example.
1. (15 points) Express each of these statements using quantifiers. Then form the negation of the statement so that no negation is to the left of a quantifier. Next, express the negation in simple English. (Do not simply use the phrase “It is not the case that.”)

(a) The file system cannot be backed up if there is a user currently logged on.
(b) There are at least two paths connecting every two distinct endpoints on the network.
(c) No one knows the password of every user on the system except for the system administrator, who knows all passwords.

(Similar to Rosen 1.4 # 33, 40; 1.5 # 18)

2. (5 points) As mentioned in the textbook, the notation \( \exists! x P(x) \) denotes “There exists a unique \( x \) such that \( P(x) \) is true.” If the domain consists of all integers, what is the truth value of the statement

\[ \exists! x (x + 3 = 2x) \]

Justify your answer. (Parts of Rosen 1.4 # 52)

3. (10 points) Express the quantification \( \exists! x P(x) \) using universal quantifications, existential quantifications, and logical operators.

(Rosen 1.5 # 52)

4. (10 points) Determine the truth value of each of these statements if the domain of each variable consists of all real numbers. Justify your answers.

(a) \( \exists x \forall y (xy = 0) \).
(b) \( \exists x \exists y (x + y \neq y + x) \).
(c) \( \exists x \forall y (y \neq 0 \rightarrow xy = 1) \).

(Similar to Rosen 1.5 # 28)

5. (10 points) Show that the propositions \( p_1, p_2, p_3, \) and \( p_4 \), can be shown to be logically equivalent by showing that \( p_1 \leftrightarrow p_4, \) \( p_2 \leftrightarrow p_3, \) and \( p_1 \leftrightarrow p_3 \) are each tautologies.

(Rosen 1.7 # 36)