CSE 140: Components and Design Techniques for Digital Systems

Lecture 6: Universal Gates

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Combinational Logic: Other Types of Gates

- Universal Set of Gates
  - Motivation, Definition, Examples

- Other Types of Gates
  1) XOR
  2) NAND / NOR
  3) Block Diagram Transfers
Universal Set of Gates: Motivation

- AND, OR, NOT: Logic gates related to reasoning from Aristotle (384-322BCE)
- NAND, NOR: Inverted AND, Inverted OR gates. For VLSI technologies, all gates are inverted (AND, OR operation with a bubble at output).
- Multiplexer + input table: Table based logic for programmability. FPGA technology.
- XOR: Exclusive OR gates. Parity check.

In the future, we may have new sets of gates due to new technologies. Given a set of gates, can the gates in the set cover all possible switching functions?
Universal Set

Universal set is a powerful concept to identify the coverage of a set of gates afforded by a given technology.

Criterion: If the set of gates can implement AND, OR, and NOT gates, the set is universal.
Universal Set Definition

Universal Set: A set of gates such that every switching function can be implemented with the gates in this set.

Examples

{AND, OR, NOT}  \( \text{Yes} \)  \( (a' \cdot b')' = a + b \)
{AND, NOT}  \( \text{Yes} \)  \( \text{OR gate} \)
{OR, NOT}  \( \text{Yes} \)  \( (a' + b')' = a \cdot b \)
Universal Set

Universal Set: A set of gates such that every switching function can be implemented with the gates in this set.

Examples

{AND, OR, NOT} OR can be implemented with AND & NOT gates: $a + b = (a' \cdot b')'$

{OR, NOT} AND can be implemented with OR & NOT gates: $a \cdot b = (a' + b')'$

{XOR} is not universal

{XOR, AND} is universal
iClicker

Is the set \{AND, OR\} (but no NOT gate) universal?
A. Yes
B. No

\{XOR, OR\} universal?

XOR \Rightarrow \text{NOT}
iClicker

Is the set \{f(x,y)=xy'\} universal?

A. Yes
B. No

\[
x = 1 \\
y \\
x = 1 \\
y' = y' \\
\text{AND} \left\{ \begin{array}{l} \text{AND} \\
\text{NOT} \end{array} \right\} \text{OR}
\]

\[
x \rightarrow \boxed{\text{OR}} \rightarrow x(y')' = xy
\]
\{\text{NAND, NOR}\}

\{\text{XOR}\}

\{\text{XOR, AND}\}
XOR

A ⊕ B = A'B + B'A

(A ⊕ B)' = A'B' + AB

Universal?

A. Yes
B. No

A + B

\{ XOR, XNOR \} Universal?

A. Yes
B. No
Other Types of Gates: Properties and Usage

1) XOR  \[ X \oplus Y = XY' + X'Y \]

It is a parity function (examples) useful for testing because the flipping of a single input changes the output.

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<th>id</th>
<th>x</th>
<th>y</th>
<th>x (\oplus) y</th>
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<table>
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<th>x=0</th>
<th>x=1</th>
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<tbody>
<tr>
<td>y=0</td>
<td>0</td>
</tr>
<tr>
<td>y=1</td>
<td>1</td>
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</tbody>
</table>
Other Types of Gates: Properties and Usage

1) XOR \( X \oplus Y = XY' + X'Y \)

(a) Commutative \( X \oplus Y = Y \oplus X \)

(b) Associative \( (X \oplus Y) \oplus Z = X \oplus (Y \oplus Z) \)

(c) \( 1 \oplus X = X' \), \( 0 \oplus X = 0 \oplus X = X' \)

(d) \( X \oplus X = 0 \), \( X \oplus X' = 1 \)

\( 1 \oplus 1 = \overline{X} \oplus \overline{X} = \overline{X} + X = 1 \)

\( 0 \oplus 0 = \overline{X} \oplus \overline{X} = \overline{X} + X = 0 \)
e) If \( ab = 0 \), then \( a \oplus b = a + b \)

Proof: If \( ab = 0 \), then 
\[
a = a (b + b') = ab + ab' = ab'
\]
\[
b = b (a + a') = ba + ba' = a' b
\]
\[
a + b = ab' + a' b = a \oplus b
\]

f) \( f(x,y) = x \oplus xy' \oplus x'y \oplus (x + y) \oplus x = ? \)

(Priority of operations: AND, \( \oplus \), OR)

Hint: We apply Shannon’s Expansion.
Shannon’s Expansion (for switching functions)

Formula: \( f(X,Y) = X \times f(1,Y) + X' \times f(0,Y) \)

Proof by enumeration:
If \( X = 1 \), \( f(X,Y) = f(1,Y) : 1 \times f(1,Y) + 1' \times f(0,Y) = f(1,Y) \)

If \( X = 0 \), \( f(X,Y) = f(0,Y) : 0 \times f(1,Y) + 0' \times f(0,Y) = f(0,Y) \)
Other types of gates: XOR

Simplify the function (Priority of operations: AND, $\oplus$, OR)

\[ f(X, Y) = X \oplus XY' \oplus X' Y \oplus (X+Y) \oplus X \]

Case $X = 1$: \( f (1, Y) = 1 \oplus Y' \oplus 0 \oplus 1 \oplus 1 = Y \)

Case $X = 0$: \( f (0, Y) = 0 \oplus 0 \oplus Y \oplus Y \oplus 0 = 0 \)

Thus, using Shannon’s expansion, we have

\[ f (X, Y) = Xf(1,Y)+X'f(0,Y) = XY \]

\[ xy + x'y' = xy \]
\[ f(a, b, c) = a \oplus ab \oplus (a+b) \oplus bc \]

Which variable to split the function

A. a
B. b
C. c
D. none
XOR gates

iClicker: Is the equation

\[ a + (b \oplus c) = (a+b) \oplus (a+c) \]

true?

A. Yes
B. No
2) NAND, NOR gates

NAND (NOR) gates are not associative

Let $a \mid b = (ab)'$

$(a \mid b) \mid c \neq a \mid (b \mid c)$

$[(ab)'c]' = a' + (bc)$

$ab + c'$
Other Types of Gates: Block Diagram Transform

3) Block Diagram Transformation

a) Reduce # of inputs.
b. DeMorgan’s Law

\[(a+b)' = a' \cdot b'\]

\[(ab)' = a' + b'\]
Other Types of Gates: Block Diagram Transform

c. Sum of Products (Using only NAND gates)

Sum of Products

Two-level NAND

Sum of Products (We create many bubbles with NOR gates)

Two-level NOR gates
d. Product of Sums (NOR gates only)

We will create many bubbles with NAND gates.

Sum of Products
Remark:
Two level NAND gates: Sum of Products
Two level NOR gates: Product of Sums
Part II. Sequential Networks

Flip flops
Specification
Implementation