CSE 140
Discussion Session #1
Topics

✓ Consensus Theorem
✓ Shannon’s Expansion
✓ Truth Table, Canonical Forms and Combinational Circuits
Consensus Theorem

- In Sum of Products (SOP) Form
  - $AB + B'C + AC = AB + B'C$

- In Product of Sums (POS) Form
Proof of Consensus Theorem (POS)

\[ (A + B)(B' + C)(A + C) = (A + B)(B' + C) \]

L.H.S. = \((A + B)(B' + C)(A + C)\)

\[ = (A + B)(B' + C)(A + C + 0) \]  \hspace{5mm} // (Identity)
\[ = (A + B)(B' + C)(A + BB' + C) \]  \hspace{5mm} // Complements
\[ = (A + B)(B' + C) (A + B + C)(A + B' + C) \]  \hspace{5mm} // Distributive
\[ = (A + B) (A + B + C) (B' + C) (A + B' + C) \]  \hspace{5mm} // Commutative
\[ = (A + B) (1 + C) (B' + C) (A + 1) \]  \hspace{5mm} // Distributive
\[ = (A + B) (1) (B' + C) (1) \]  \hspace{5mm} // Identity
\[ = (A + B)(B' + C) \]

Consensus Theorem Visualization using Truth Table (POS)

- \(F(A,B,C) = (A + B)(B' + C)(A + C)\)
- \(G(A,B,C) = (A + B)(B' + C)\)

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<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>A + B</th>
<th>B' + C</th>
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Consensus Theorem Examples

In Sum of Products Form
• $AB + B'CD' = AB + B'CD' + ACD'$
• $AC'D + BCE = AC'D + BCE + ABDE$

In Product of Sums Form
• $(A + B)(B' + C + D') = (A + B)(B' + C + D')(A + C + D')$
• $(A + B + C')(C + D + E') = (A + B + C')(C + D + E')(A + B + D + E')$
Shannon’s Expansion

\[ f(A,B,C) = A \cdot f(1,B,C) + A' \cdot f(0,B,C) = B \cdot f(A,1,C) + B' \cdot f(A,0,C) \]

\[ f(A,B,C) = A \cdot f(1,B,C) + A' \cdot f(0,B,C) \]
\[ = A \cdot (B \cdot f(1,1,C) + B' \cdot f(1,0,C)) + A' \cdot (B \cdot f(0,1,C) + B' \cdot f(0,0,C)) \]
\[ = AB \cdot f(1,1,C) + AB' \cdot f(1,0,C) + A'B \cdot f(0,1,C) + A'B' \cdot f(0,0,C) \]

\[ f(A,B,C) = (A + f(0,B,C)) \cdot (A' + f(1,B,C)) = (B + f(A,0,C)) \cdot (B' + f(A,1,C)) \]

\[ f(A,B,C) = (A + f(0,B,C)) \cdot (A' + f(1,B,C)) \]
\[ = (A + (B + f(0,0,C)) \cdot (B' + f(0,1,C))) \cdot (A' + (B + f(1,0,C)) \cdot (B' + f(1,1,C))) \]
\[ = (A + B + f(0,0,C)) \cdot (A + B' + f(0,1,C)) \cdot (A' + B + f(1,0,C)) \cdot (A' + B' + f(1,1,C)) \]
Consensus Using Shannon’s Expansion (SOP)

- \( AB + B'C + AC = AB + B'C \)
  
  \( f(A,B,C) = AB + B'C + AC \)
  
  \[ f(A,B,C) = B.f(A,1,C) + B'.f(A,0,C) \]

  \[ f(A,1,C) = A.1 + 0.C + AC = A + AC = A(1+C) = A \]
  
  \[ f(A,0,C) = A.0 + 1.C + AC = C + AC = C(1+A) = C \]

  \[ f(A,B,C) = B.f(A,1,C) + B'.f(A,0,C) \]
  
  \[ = B.A + B'C \]
  
  \[ = AB + B'C \]

- \( f(A,B,C) = AB + B'C + AC = AB + B'C \)
Consensus Using Shannon’s Expansion (POS)


\[f(A, B, C) = (A + B)(B' + C)(A + C)\]

\[f(A, B, C) = (B + f(A, 0, C))(B' + f(A, 1, C))\]

\[f(A, 0, C) = (A + 0)(1 + C)(A + C) = (A)(1)(A + C) = A + AC = A(1+C) = A\]

\[f(A, 1, C) = (A + 1)(0 + C)(A + C) = (1)(C)(A + C) = AC + C = C(A+1) = C\]

\[f(A, B, C) = (B + f(A, 0, C))(B' + f(A, 1, C))\]

\[= (B + A)(B' + C)\]

\[= (A + B)(B' + C)\]

A machine inputs 3 binary bits \((X_2,X_1,X_0)\) and outputs \(Y=1\) when the number of bits are less than or equal to 1. Otherwise, the output is \(Y=0\).
## Truth Tables, Canonical Forms and Circuits

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<th>X1</th>
<th>X0</th>
<th>Y</th>
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</table>

- **Sum of Products**
  \[
f(X_2, X_1, X_0) = X_2'.X_1'.X_0' + X_2'.X_1'.X_0 + X_2'.X_1.X_0' + X_2.X_1'.X_0'
  = \Sigma m(0,1,2,4)
  \]

- **Product of Sums**
  \[
f(X_2, X_1, X_0) = (X_2 + X_1' + X_0')(X_2' + X_1 + X_0')(X_2' + X_1' + X_0)(X_2' + X_1' + X_0')
  = \Pi M(3,5,6,7)
  \]

\[
f(X_2, X_1, X_0) = X_2'.X_1'.X_0' + X_2'.X_1'.X_0 + X_2'.X_1.X_0' + X_2'.X_1'.X_0' + X_2'.X_1'.X_0' + X_2'.X_1'.X_0' + X_2'.X_1'.X_0'
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  = X_0'X_1' + X_0'X_2 + X_0'X_2'
  = X_0'X_1' + X_0'X_2 + X_0'X_2'
  = X_0'X_1' + X_0'X_2 + X_0'X_2'
\]
Truth Tables, Canonical Forms and Circuits

[Diagram of a circuit with inputs X2, X1, and X0]
Truth Tables, Canonical Forms and Circuits

\[
f(X_2, X_1, X_0) = X_0'X_1' + X_0'X_2' + X_1'X_2'
\]

<table>
<thead>
<tr>
<th>Variables</th>
<th>3 (X0,X1,X2)</th>
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<tbody>
<tr>
<td>Literals</td>
<td>3 (X0',X1',X2')</td>
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<tr>
<td>Operators</td>
<td>11 (3 AND, 2 OR, 6 NOT)</td>
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<tr>
<td>Gates</td>
<td>7 (3 AND, 1 OR, 3 NOT)</td>
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<td>Nets</td>
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<td>Pins</td>
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Thank You!

- Remember:
  - Homework 1 due on Tuesday 1/12 11:59 PM