Problem 1  Consider the following state diagram for a Turing machine with input alphabet \( \Sigma = \{1\} \) and tape alphabet \( \Gamma = \{1, x, y, \_\} \). Any transition not drawn is to \( q_r \).

What is the language of this Turing machine? Explain how the machine works.

Problem 2  Consider a variant definition of Turing machines, called always-right Turing machines. In such a machine, the transition function \( \delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{ R \} \) specifies that the machine always moves its head right ("R"). There is no way for a machine in the always-right model to move its head left.

Show that the always-right Turing machine model is not equivalent to the Turing machine model defined in class and in Sipser.

**Hint:** Always-right Turing machines are, in fact, equivalent in power to DFAs.

Problem 3  Prove that the class of Turing-recognizable (aka recursively enumerable) languages is closed under concatenation.