CSE 21
Lecture 3: Graphs
Course Website

- Course details is available online: http://cseweb.ucsd.edu/classes/wi14/cse21-a/
- Regular announcements will be made on the webpage
- First Assignment is uploaded
- Solutions to the assignment will be uploaded on Monday
- Discussion forum set up in Piazza
There will be around 6 quizzes (5% each)
Most of them will be in class.
Some online quizzes will be given on WeBWork.
Total marks for online quizzes will be 5%.
First quiz will be on Wednesday (15th January) in class.
Representation of data

- Numbers - represented over any base. In particular one can represent a number over binary.
- Sets - binary vectors.
- Functions - Truth table.
- Relationship among elements -
Representation of relation among elements

- Vertices - set of elements.

\[ V = \{v_1, \ldots, v_n\} \]
Representation of relation among elements

- Vertices - set of elements.
  \[ V = \{v_1, \ldots, v_n\} \]

- Edges - set of pairs of vertices.
Vertices - set of elements.

\[ V = \{v_1, \ldots, v_n\} \]

Edges - set of pairs of vertices.

\[ E = \{e_1, \ldots, e_m\} \]

Given the set of vertices and edges we have a graph

\[ G = (V, E) \]
Let $G = (V, E)$ be a graph.
Basic Definitions

- Let $G = (V, E)$ be a graph.
- If $(u, v) \in E$ implies $(v, u) \in E$ then it is called an undirected graph.
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- If $(u, v) \in E$ implies $(v, u) \in E$ then it is called an undirected graph.
- An weight can be assigned to each edge. In that case it is called an weighted graph.
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- Let $G = (V, E)$ be a graph.
- If there is an edge from vertex $u$ to $v$ we say $v$ is a neighbor of $u$.
- For an undirected graph the total number of $u$ such that $(u, v) \in E$ is called degree of $v$. 
Example of a graph

Vertices

- a
- b
- c
- d
- e
- f
- g
Example of a graph

Undirected Graph
Example of a graph

Weighted Undirected Graphs

- $a$ connected to $b$ with weight 7
- $b$ connected to $c$ with weight 8
- $b$ connected to $d$ with weight 9
- $b$ connected to $e$ with weight 7
- $d$ connected to $e$ with weight 15
- $e$ connected to $f$ with weight 8
- $e$ connected to $g$ with weight 9
- $f$ connected to $g$ with weight 11
Example of a graph

Weighted Directed Graphs

- **Nodes:** a, b, c, d, e, f, g
- **Edges and Weights:**
  - a → b: 7
  - b → c: 8
  - b → d: 9
  - d → b: 15
  - d → e: 6
  - e → d: 5
  - e → f: 8
  - e → g: 9
  - f → e: 8
  - f → g: 11
How many edges can be there is a simple undirected graph on \( n \) vertices?
Advantages of a graph

- Mathematical way of expressing relations among objects.
- Very simple.
- Very general.
Example: Friendship graph

- Each person is a vertex.
- If two people are friends then there is an edge between the respective vertices.

Used for understanding social networks, like Facebook.
Example: Friendship graph
Example: Internet Graph

- Every website is a vertex.
- If a website has a link to another website then there is a directed edge from the first vertex to the second.

Used for web crawls by Google.
Example: Internet Graph
Example: Road Network

- Vertices are the cities
- Edges are the roads
Example: Road Network

[Diagram of a road network with cities such as Amsterdam, Brussels, Paris, Geneva, Frankfurt, Munich, Milan, and Florence, connected by lines with distances labeled.]
Other Examples

- Many other problems in real life can be designed as a problem in graph theory.
- So studying the structure of graphs and designing algorithms for graph problems is an important field.
In a room there are $2n$ people. Some of the people shake hand with each other in such a way that if persons $A$ and $B$ shake hand and persons $B$ and $C$ shake hand then person $A$ and $C$ does not shake hand.

What is the maximum number of handshakes possible in this case?
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What is the maximum number of handshakes possible in this case?

Ans: $n^2$