1. The triangle inequality says that for any two real numbers $x$ and $y$,

$$|x + y| \leq |x| + |y|$$

. Show that for any $n$ real numbers $x_1, x_2, \ldots, x_n$ we have

$$|x_1 + \cdots + x_n| \leq |x_1| + |x_2| + \cdots + |x_n|.$$ 

2. If every two cities in state A are joined by a one-way road, then it is possible to find a starting city $A$ and a route from $A$ that passes through every city exactly once.

3. In a party there are $n$ people $p_1, \ldots, p_n$. Some of them shake hand with each each other. So at the end the person $p_1$ shakes hand with $d_1$ number of other people in the party. Similarly $p_2$ shakes hand with $d_2$ number of other people in the party. Likewise for all $1 \leq i \leq n$ the person $p_i$ shakes hand with $d_i$ number of other people in the party. At the end the total number of handshakes done is $m$. Show that

$$2m = (d_1 + d_2 + \cdots + d_n)$$

4. How many edges are there is an undirected graph with 1000 edges.

5. How many distinct kind of graphs on 10 vertices are there?

6. (AM-GM Inequality) [Hard Problem] Prove that for any postive numbers $a_1, \ldots, a_n$ the following inequality is satisfied:

$$\frac{a_1 + a_2 + \cdots + a_n}{n} \geq (a_1 \times a_2 \times \cdots \times a_n)^{1/n}$$