CSE 21: Assignment Set 2

1. Prove that

\[
\sum_{i=0}^{k-1} ab^i = a \frac{b^k - 1}{b - 1}
\]

2. Prove that

\[
\sum_{i=0}^{k} (a + ib) = \frac{2a + kb}{2}(k + 1).
\]

3. Let \( n > 1 \) be an integer. In a football league there are \( n \) teams. Every two teams have played against each other exactly once, and in match no draw is allowed. Prove that it is possible to number the teams in such a way that team \( i \) beats \((i + 1)\) for \( i = 1, 2, \ldots, n - 1 \).

4. Prove that \( 2002^{n+2} + 2003^{2n+1} \) is divisible by 4005.

5. The Fibonacci sequence is defined as \( x_0 = 0, x_1 = 1 \) and \( x_{n+2} = x_{n+1} + x_n \) for all non-negative integers \( n \). Prove that

   (a) \( x_m = x_{r+1}x_{m-r} + x_rx_{m-r-1} \) for all integers \( m \geq 1 \) and \( 0 \leq r \leq m - 1 \);

   (b) \( x_d \) divides \( x_{kd} \) for all positive integers \( d \) and \( k \).

6. (Hard Problem) For natural number \( p \) and \( q \), the Ramsey number \( R(p, q) \) is defined as the smallest integer \( n \) so that among any \( n \) people, there exist \( p \) of them who know each other, or there exist \( q \) of them who don’t know each other. Prove that Note that \( R(p, 1) = R(1, q) = 1 \). Prove that:

   (a) \( R(p + 1, q + 1) \leq R(p, q + 1) + R(p + 1, q) \)

   (b) \( R(p, q) \leq C_p^{p+q-2} \)