CSE 21: Assignment Set 1

1. Prove that there is an unique representation of any positive integer in any base \( b \). State the problem in mathematical terms and then prove it.

2. What is the maximum integer that can be represented in base 2 using only 10 bits (that is, what is the largest integer which when represented in base 2 has at most length 10 representation).

3. Write down the truth table of the function \( f : \{0, 1\}^3 \to \{0, 1\} \), where

\[
f(x_1, x_2, x_3) = (x_1 \lor x_2) \land x_3.
\]

4. Let \( x > -1 \) be a real number. Prove that \( (1 + x)^n \geq 1 + nx \) for all natural numbers \( n \).

5. Prove that \( \sum_{i=1}^{n} i \times i! = (n + 1)! - 1 \).

6. Let \( \{a_n\} \) be a sequence of natural numbers such that \( a_1 = 5, a_2 = 13 \) and \( a_{n+2} = 5a_{n+1} - 6a_n \) for all natural numbers \( n \). Prove that \( a_n = 2^n + 3^n \) for all natural number \( n \).

7. Let \( n > 1 \) be an integer. In a football league there are \( n \) teams. Every two teams have played against each other exactly once, and in match no draw is allowed. Prove that it is possible to number the teams in such a way that team \( i \) beats \( (i + 1) \) for \( i = 1, 2, \ldots, n - 1 \).