CSE 20
Lecture 15: Midterm Review
Midterm Review

- Representation of integers in base $b$
- Propositional Logic
- Proof systems:
  - Direct Proof
  - Proof by contradiction
  - Contrapositive
- Sets Theory
- Functions
Representation of integers in base $b$

- Every integer $n$ can be represented in base $b$
- In base $b$ the digits are $\{0, 1, \ldots, b - 1\}$
- Any $n$ can be written in an unique way as
  \[x_0 b^0 + x_1 b^1 + \cdots + x_t b^t.\]
- $n$ is written as $[x_0 x_1 \ldots x_t]_b$
Every statement is either TRUE or FALSE

There are logical connectives $\lor$, $\land$, $\neg$, $\implies$ and $\iff$.

Two logical statements can be equivalent if the two statements answer exactly in the same way on every input.

To check whether two logical statements are equivalent one can do one of the following:

- Checking the Truthtable of each statement
- Reducing one to the other using reductions
Propositional Logic Rules

- **Associative Law:** 
  
  \[ ((p \lor q) \lor r) = (p \lor (q \lor r)) \] 
  and 
  
  \[ ((p \land q) \land r) = (p \land (q \land r)) \]

- **Commutative Law:** 
  
  \[ (p \lor q) = (q \lor p) \] 
  and 
  
  \[ (p \land q) = (q \land p) \]

- **Distributive Law:** 
  
  \[ p \lor (q \land r) = (p \lor q) \land (p \lor r) \] 
  and 
  
  \[ p \land (q \lor r) = (p \land q) \lor (p \land r) \]

- **De Morgan’s Law:** 
  
  \[ \neg(p \lor q) = (\neg p \land \neg q) \] 
  and 
  
  \[ \neg(p \land q) = (\neg p \lor \neg q) \]
If we want to write the expression \( p \implies q \) using \( \lor \), \( \land \) and \( \neg \), what is the answer:

\[ (p \lor q) \]
\[ (p \land q) \]
\[ (\neg p \lor q) \]
\[ (\neg p \land q) \]
\[ (p \lor \neg q) \]
iclicker question

If we want to write the expression $p \implies q$ using $\lor, \land$ and $\neg$ what is the answer:
If we want to write the expression $p \implies q$ using $\lor, \land$ and $\neg$ what is the answer:

- $(p \lor q)$
- $(p \land q)$
- $(\neg p \lor q)$
- $(\neg p \land q)$
- $(p \lor \neg q)$
iclicker question

If we want to write the expression $p \implies q$ using $\lor$, $\land$ and $\neg$ what is the answer:

- $(p \lor q)$
- $(p \land q)$
- $(\neg p \lor q)$
- $(\neg p \land q)$
- $(p \lor \neg q)$
There are two important symbols: \( \forall \) and \( \exists \).

Some statements can be defined using a variable.

For example: \( P_x = "4x^2 + 3 \) is divisible by 5"

We can have statements like: \( \forall x \in \mathbb{Z}, 4x^2 + 3 \) is divisible by 5.

Or \( \exists x \in \mathbb{Z}, 4x^2 + 3 \) is divisible by 5.
Rules of negation

\[ \neg (\forall x, P_x) = (\exists x, \neg P_x) \]
\[ \neg (\exists x, P_x) = (\forall x, \neg P_x) \]
What is the negation of the statement "all human beings can walk on two legs and think logically"?

1. No human being can walk on two legs and cannot think logically
2. No human being can walk on two legs or cannot think logically
3. There exists a human being who can't walk on two legs or cannot think logically
4. There exists a human being who can't walk on two legs and cannot think logically
5. If someone cannot walk on two legs or cannot think logically then it is not a human being.
iclicker question

What is the negation of the statement “all human beings can walk on two legs and think logically”
What is the negation of the statement “all human beings can walk on two legs and think logically”

- No human being can walk on two legs and cannot think logically
- No human being can walk on two legs or cannot think logically
- There exists a human being who can’t walk on two legs or cannot think logically
- There exists a human being who can’t walk on two legs and cannot think logically
- If someone cannot walk on two legs or cannot think logically then it is not a human being.
What is the negation of the statement “all human beings can walk on two legs and think logically”

- No human being can walk on two legs and cannot think logically
- No human being can walk on two legs or cannot think logically
- There exists a human being who can’t walk on two legs or cannot think logically
- There exists a human being who can’t walk on two legs and cannot think logically
- If someone cannot walk on two legs or cannot think logically then it is not a human being.
Proof Techniques

If one has to prove $statementA \implies statementB$ then one of the following steps can be adapted:

- Directly prove $A \implies B$
- Prove that $(\neg B \land A) = False$ (Or gives a contradiction)
If one has to prove \( \text{statement} A \implies \text{statement} B \) then one of the following steps can be adapted:

- Directly prove \( A \implies B \)
- Prove that \( (\neg B \land A) = \text{False} \) (Or gives a contradiction)
- To prove \( (\neg B \land A) = \text{False} \) one can prove \( (\neg B \land A) \implies \text{False} \)
Proof Techniques

If one has to prove \( \text{statement} A \implies \text{statement} B \) then one of the following steps can be adapted:

- Directly prove \( A \implies B \)
- Prove that \( (\neg B \land A) = False \) (Or gives a contradiction)
- To prove \( (\neg B \land A) = False \) one can prove \( (\neg B \land A) \implies False \)
- \( \neg B \implies \neg A \).
To prove the following statement by contradiction what needs to be proved:

\[ \forall x \in \mathbb{Z}, \text{if } x \text{ is even and } x^3 - x \text{ is not divisible by 6 then we get a contradiction.} \]

If there exists \( x \in \mathbb{Z} \) such that \( x \) is even and \( x^3 - x \) is not divisible by 6 then we get a contradiction.

\[ \forall x \in \mathbb{Z}, \text{if } x \text{ is not even then } x^3 - x \text{ is not divisible by 6} \]

If there exists \( x \in \mathbb{Z} \) such that \( x \) is not even or \( x^3 - x \) is not divisible by 6 then we get a contradiction.
To prove the following statement by contradiction what needs to be proved:

“\( \forall x \in \mathbb{Z}, x \text{ is even then } x^3 - x \text{ is divisible by 6} \)”
To prove the following statement by contradiction what needs to be proved:

“∀ \( x \in \mathbb{Z} \), \( x \) is even then \( x^3 - x \) is divisible by 6”

- \( \forall x \in \mathbb{Z} \), if \( x \) is even and \( x^3 - x \) is not divisible by 6 then we get a contradiction
- If \( \exists x \in \mathbb{Z} \) such that \( x \) is even and \( x^3 - x \) is not divisible by 6 then we get a contradiction
- If there exists \( x \in \mathbb{Z} \) such that \( x^3 - x \) is not divisible by 6 and \( x \) is not even then we have a contradiction.
- \( \forall x \in \mathbb{Z} \), \( x \) is not even then \( x^3 - x \) is not divisible by 6
- If \( \exists x \in \mathbb{Z} \) such that \( x \) is not even OR \( x^3 - x \) is not divisible by 6 then we get a contradiction.
To prove the following statement by contradiction what needs to be proved:

“∀x ∈ ℤ, x is even then x^3 − x is divisible by 6”

- ∀x ∈ ℤ, if x is even and x^3 − x is not divisible by 6 then we get a contradiction
- If ∃x ∈ ℤ such that x is even and x^3 − x is not divisible by 6 then we get a contradiction
- If there exists x ∈ ℤ such that x^3 − x is not divisible by 6 and x is not even then we have a contradiction.
- ∀x ∈ ℤ, x is not even then x^3 − x is not divisible by 6
- If ∃x ∈ ℤ such that x is not even OR x^3 − x is not divisible by 6 then we get a contradiction.