Announcements

- Read Trucco & Verri: pp. 22-40
- Irfanview: http://www.irfanview.com/ is a good Windows utility for manipulating images. Try xv for linux.

Pinhole Camera: Perspective projection

- Abstract camera model - box with a small hole in it

The equation of projection

Cartesian coordinates:
- We have, by similar triangles, that 
  \((x, y, z) \rightarrow (f x/z, f y/z, -f)\)
- Ignoring the third coordinate, we get 
  \((x, y, z) \rightarrow (f x/z, f y/z)\)

Euclidean -&gt; Homogenous -&gt; Euclidean

In 2-D
- Euclidean -&gt; Homogenous: 
  \((x, y) \rightarrow \lambda (x, y, 1)\)  
  (can just take \(\lambda = 1\))
- Homogenous -&gt; Euclidean: 
  \((x, y, z) \rightarrow (x/z, y/z)\)

In 3-D
- Euclidean -&gt; Homogenous: 
  \((x, y, z) \rightarrow \lambda (x, y, z, 1)\)  
  (can just take \(\lambda = 1\))
- Homogenous -&gt; Euclidean: 
  \((x, y, z, w) \rightarrow (x/w, y/w, z/w)\)
The camera matrix

Turn

\[ (x, y, z) \rightarrow (x/z, y/z) \]

into homogenous coordinates

– HC’s for 3D point are (X, Y, Z, 1)
– HC’s for point in image are (U, V, W)

\[
\begin{bmatrix}
X \\
Y \\
Z \\
1
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}
\]

Affine Camera Model

- Take Perspective projection equation, and perform Taylor Series Expansion about some point \((x_0, y_0, z_0)\).
- Drop terms of higher order than linear.
- Resulting expression is called affine camera model.

\[
\begin{bmatrix}
u \\
v \\
w
\end{bmatrix} =
\begin{bmatrix}
1/\tilde{z}_0 & 0 & -x_0/\tilde{z}_0^2 \\
0 & 1/\tilde{z}_0 & -y_0/\tilde{z}_0^2 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} = A \cdot p + b
\]

- Properties
  – Points map to points
  – Lines map to lines
  – Parallel lines map to parallel lines (no vanishing point – at infinity)
  – Ratios of distance/angles preserved

Orthographic projection

Start with affine camera model, and take Taylor series about \((x_0, y_0, z_0) = (0, 0, z_0)\) – a point on optical axis

\[
u = f/\tilde{z}_0 \]
\[v = f/\tilde{z}_0 \]

Depth (z) is lost

Three projection models

- Perspective – all depths
- Affine – points near point of Taylor series expansion. E.g., when depth and size variation is small, and camera is far from point relative to size.
- Orthographic – when object is near optical axis, and depth variation of object is small compared to distance to the object.

What if camera coordinate system differs from object coordinate system?

Euclidean Coordinate Systems

\[
x = \overrightarrow{OP}_x = \overrightarrow{O} + xi + yj + zk
\]
\[
y = \overrightarrow{OP}_y = \overrightarrow{O}
\]
\[
z = \overrightarrow{OP}_z = \overrightarrow{O}
\]

\[
P = \begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
\]
Coordinate Changes: Pure Translations
No rotation (e.g., \( i_A \rightarrow i_B \) etc)

\[
\mathcal{O}_B P = \mathcal{O}_B O_A + \mathcal{O}_A P, \quad B P = A P + B O_A
\]

Coordinate Changes: Pure Rotations

\[
\begin{bmatrix}
    i_B \\
    j_B \\
    k_B
\end{bmatrix}
= R
\begin{bmatrix}
    i_A \\
    j_A \\
    k_A
\end{bmatrix}
\]

\[
\begin{bmatrix}
    i_B \\
    j_B \\
    k_B
\end{bmatrix}
= R
\begin{bmatrix}
    i_A \\
    j_A \\
    k_A
\end{bmatrix}
\]

Rotation Matrix

\[
\begin{bmatrix}
    i_A \\
    j_A \\
    k_A
\end{bmatrix}
= \begin{bmatrix}
    i_B \\
    j_B \\
    k_B
\end{bmatrix}
\]

\[
\begin{bmatrix}
    i_A \\
    j_A \\
    k_A
\end{bmatrix}
= \begin{bmatrix}
    i_B \\
    j_B \\
    k_B
\end{bmatrix}
\]

Coordinate Changes: Rigid Transformations
Rotation + Translation

\[
B P = B A R A P + B O_A
\]

More about rotations matrices

A rotation matrix \( R \) has the following properties:

- Its inverse is equal to its transpose \( R^{-1} = R^T \) or
  \( R^T R = I \)
- Its determinant is equal to 1: \( \det(R) = 1 \).

Or equivalently:

- Rows (or columns) of \( R \) form a right-handed orthonormal coordinate system.
- Even though a rotation matrix is 3x3 with nine numbers, it only has three degrees of freedom, it can be parameterized with three numbers.
- There are many parameterizations.
Homogeneous Representation of Rigid Transformations

\[
\begin{bmatrix}
    \mathbf{p}' \\
    1
\end{bmatrix} = \begin{bmatrix}
    \mathbf{R} & \mathbf{t} \\
    0 & 1
\end{bmatrix} \begin{bmatrix}
    \mathbf{p} \\
    1
\end{bmatrix} = \begin{bmatrix}
    \mathbf{R} \mathbf{p} + \mathbf{t} \\
    0 & 1
\end{bmatrix}
\]

Transformation represented by 4 by 4 Matrix

Block Matrix Multiplication

Given

\[
A = \begin{bmatrix}
    a_{11} & a_{12} \\
    a_{21} & a_{22}
\end{bmatrix},
B = \begin{bmatrix}
    b_{11} & b_{12} \\
    b_{21} & b_{22}
\end{bmatrix}
\]

What is \(AB\) ?

\[
AB = \begin{bmatrix}
    a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\
    a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22}
\end{bmatrix}
\]

What if camera coordinate system differs from object coordinate system?

\[
\begin{pmatrix}
    \hat{e} \\
    1
\end{pmatrix}
\]

\[
\begin{pmatrix}
    \hat{O}' \\
    1
\end{pmatrix}
\]

\[
\begin{pmatrix}
    \mathbf{R} \\
    0 \\
    1
\end{pmatrix}
\]

\[
\begin{pmatrix}
    \mathbf{W}' \\
    1
\end{pmatrix}
\]
Intrinsic parameters

- 3x3 homogenous matrix
- Focal length:
- Principal Point: C
- Orientation and position of image coordinate system
- Pixel Aspect ratio

Camera parameters

- Extrinsic Parameters: Since the camera may not be at the origin, there is a rigid transformation between the world coordinates and the camera coordinates
- Intrinsic parameters: Since scene units (e.g., cm) differ image units (e.g., pixels) and coordinate system may not be centered in image, we capture that with a 3x3 transformation comprised of focal length, principal point, pixel aspect ratio, angle between axes, etc.

\[
\begin{pmatrix}
U \\
V \\
W
\end{pmatrix} = \text{Transformation represented by} \begin{pmatrix} 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \end{pmatrix} 
\]

\[
\begin{pmatrix}
X \\
Y \\
Z
\end{pmatrix} = \text{Rigid Transformation represented by} \begin{pmatrix} 4 & 4 \\
4 & 4 \\
4 & 4 \end{pmatrix} 
\]

What about light?

Getting more light – Bigger Aperture

Limits for pinhole cameras
Pinhole Camera Images with Variable Aperture

2 mm

.6 mm

.15 mm

1 mm

.35 mm

.07 mm

The reason for lenses