Image Formation and Cameras

Introduction to Computer Vision
CSE 152
Lecture 3

Announcements

• Assignment 0: due Today
• Sign up for Piazza: http://www.piazza.com
• Office Hour: Wednesday 2:00-3:00
• Read Trucco & Verri: pp. 15-40, 22-40
• Optional Reading. Szeliski, Chapter 2
• Scanned pages:
  22-40  97-100
  55-63  140-166
  67-81  191-198

Image Formation: Outline

• Factors in producing images
• Projection
• Perspective
• Vanishing points
• Orthographic
• Lenses
• Sensors
• Quantization/Resolution
• Illumination
• Reflectance

Earliest Surviving Photograph

• First photograph on record, “la table service” by Nicephore Niepce in 1822.
• Note: First photograph by Niepce was in 1816.

How Digital Cameras Produce Images

• Basic process:
  – photons hit a detector
  – the detector becomes charged
  – the charge is read out and digitized

• Sensor types:
  – CCD (charge-coupled device)
    • high sensitivity
    • high power
    • cannot be individually addressed
    • blooming
  – CMOS
    • most common
    • simple to fabricate (cheap)
    • lower sensitivity, lower power
    • can be individually addressed
    • Easier to integrate control/digital electronics

Images are two-dimensional patterns of brightness values.

They are formed by the projection of 3D objects.
Lighting Effects Appearance: Monet

Viewpoint affects appearance: Monet

Haystack at Chaillly at Sunrise (1865)

Weather Affects Appearance: Monet

Pinhole Camera: **Perspective projection**

- Abstract camera model - box with a small hole in it

Camera Obscura

*When images of illuminated objects ... penetrate through a small hole into a very dark room ... you will see [on the opposite wall] these objects in their proper form and color, reduced in size ... in a reversed position, owing to the intersection of the rays*.  

Leonardo da Vinci

http://www.acmi.net.au/AIC/CAMERA_OBSCURA.html (Russell Naughton)

- Used to observe eclipses (eg., Bacon, 1214-1294)
- By artists (eg., Vermeer).
Camera Obscura

Jetty at Margate England, 1898.

http://brightbytes.com/cosite/collection2.html (Jack and Beverly Wilgus)

Distant objects are smaller

Note “intersection of rays” ala Leonardo.

Geometric properties of projection

• Points go to points
• Lines go to lines
• Planes go to whole image or half-plane
• Polygons go to polygons
• Angles & distances not preserved

• Degenerate cases:
  – line through focal point yields point
  – plane through focal point yields line

Parallel lines meet in the image

• The projection of parallel lines meet at the vanishing point
• Intersection of line through O parallel to the 3-D line(s)
• A single line can have a vanishing point

Vanishing points

Different 3D directions correspond to different vanishing points

Learning to draw in perspective
Take out paper and pencil
And ruler if you’ve got one
Where are the vanishing points? Sometimes called 1-point perspective Two point perspective.

Vanishing Points

The equation of projection

Cartesian coordinates:
- We have, by similar triangles, that $(x, y, z) \rightarrow (f/x/z, f/y/z, -f)$
- Ignoring the third coordinate, we get

What is the intersection of two lines in a plane?
Do two lines in the plane always intersect at a point?

No, Parallel lines don’t meet at a point.

Can the perspective image of two parallel lines meet at a point?

YES

Homogenous coordinates

A way to represent points in a projective space

1. Add an extra coordinate
e.g., \((x,y) \rightarrow (x,y,1)=(u,v,w)\)

2. Impose equivalence relation such that \((\lambda \not= 0)\)
\((u,v,w) \approx \lambda*(u,v,w)\)
i.e., \((x,y,1) \approx (\lambda x, \lambda y, \lambda)\)

3. “Point at infinity” – zero for last coordinate
e.g., \((x,y,0)\)

• Why do this?
– Possible to write the action of a perspective camera as a matrix
– Possible to represent points “at infinity”
  • Where parallel lines intersect
  • Vanishing points are the projection of points of points at infinity

Changes of coordinates:

Euclidean -> Homogenous-> Euclidean

In 2-D
• Euclidean -> Homogenous: \((x, y) \rightarrow k (x,y,1)\)
• Homogenous -> Euclidean: \((u,v,w) \rightarrow (u/w, v/w)\)

In 3-D
• Euclidean -> Homogenous: \((x, y, z) \rightarrow k (x,y,z,1)\)
• Homogenous -> Euclidean: \((x, y, z, w) \rightarrow (x/w, y/w, z/w)\)
The camera matrix

\[(x,y,z) \rightarrow (f \frac{x}{z}, f \frac{y}{z})\]

Turn this expression into homogeneous coordinates
- HC's for 3D point are (X,Y,Z,T)
- HC's for point in image are (U,V,W)

\[
\begin{pmatrix}
U \\
V \\
W
\end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{pmatrix} \begin{pmatrix} x \\
y \\
z
\end{pmatrix}
\]

Perspective Camera Matrix
A 3x4 matrix

End of the Digression

Simplified Camera Models

- Perspective Projection
  - Assume that \( f=1 \), and perform a Taylor series expansion about \((x_0, y_0, z_0)\)
  \[
  \begin{pmatrix}
  u \\
v
\end{pmatrix} = \begin{pmatrix} 1 & \frac{x}{z} & \frac{y}{z} & \frac{0}{z}
\end{pmatrix} \begin{pmatrix} x \\
y \\
z
\end{pmatrix}
\]
  - Dropping higher order terms and regrouping,
  \[
  \begin{pmatrix} u \\
v
\end{pmatrix} = \begin{pmatrix} 1/z_0 & 0 & -x_0/z_0^2 & x_0 \\
0 & 1/z_0 & -y_0/z_0^2 & y_0
\end{pmatrix} \begin{pmatrix} x \\
y \\
z
\end{pmatrix} = Ap + b
\]

Affine Camera Model

- Take Perspective projection equation, and perform Taylor Series Expansion about some point \((x_0, y_0, z_0)\).
- Drop terms of higher order than linear.
- Resulting expression is the affine camera model

\[
\begin{pmatrix} u \\
v \\
w
\end{pmatrix} = \begin{pmatrix} 1/z_0 & 0 & -x_0/z_0^2 & x_0 \\
0 & 1/z_0 & -y_0/z_0^2 & y_0 \\
0 & 0 & 1 & 0
\end{pmatrix} \begin{pmatrix} x \\
y \\
z
\end{pmatrix}
\]

This is called the Affine Camera Model

Rewrite affine camera model in terms of homogenous coordinates

Affine Camera Matrix
Scaled Orthographic Camera Model
Consider doing an expansion about a point along the optical axis
\( X_0 = 0, Y_0 = 0 \)
\[
\begin{bmatrix}
    u \\
    v \\
    w
\end{bmatrix} =
\begin{bmatrix}
    1 / z_0 & 0 & -x_0 / z_0 \\
    0 & 1 / z_0 & -y_0 / z_0 \\
    0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    x \\
    y \\
    z
\end{bmatrix}
\]

Orthographic Camera Matrix

Unlike perspective,
• Parallel lines project to parallel lines
• Ratios of distances along a line are preserved under orthographic

Scaled Orthographic projection
Taking Taylor series about \((0, 0, z_0)\) – a point on optical axis

When are simplified models appropriate
• Orthographic projection
  – Object near optical axis
  – Object is small relative to the depth (rule of thumb, size is 1/10 of distance).
  – Zoom lens
• Affine camera model
  – Object is small relative to the depth
• Perspective
  – Other situations – e.g., wide range of depths, wide field of view, large objects