Announcements

- HW3 assigned, due March 5, 2013

Stereo Vision Outline

- Offline: Calibrate cameras & determine “epipolar geometry”
- Online
  1. Acquire stereo images
  2. Rectify images to convenient epipolar geometry
  3. Establish correspondence
  4. Estimate depth

Random Dot Stereograms

- Potential matches for $p$ have to lie on the corresponding epipolar line $l'$.
- Potential matches for $p'$ have to lie on the corresponding epipolar line $l$. 

Estimating Depth

2D world with 1-D image plane

Two measurements: $X_L$, $X_R$

Two unknowns: $X$, $Z$

Constants:
- Baseline: $d$
- Focal length: $f$

Disparity: $(X_L - X_R)$

$Z = \frac{d f}{X_L - X_R}$

$X = df \frac{X_L}{X_L - X_R}$

$X_R = f \frac{X - d}{Z}$

(Adapted from G. Hager)
Skew Symmetric Matrix & Cross Product

- The cross product $a \times b$ of two vectors $a$ and $b$ can be expressed as a matrix vector product $[a]_x b$ where $[a]_x$ is the skew symmetric matrix:

$$[a]_x = \begin{bmatrix}
0 & -a_3 & a_2 \\
 a_3 & 0 & -a_1 \\
-a_2 & a_1 & 0
\end{bmatrix}$$

- A matrix $S$ is skew symmetric iff $S = -S^T$

Epipolar Constraint: Calibrated Case

The vectors $O_p$, $O'O'$, and $O_p'$ are coplanar.

$$O_p' - (O O') \times (O_p') = 0 \quad \Rightarrow \quad p \cdot t \times (R p') = 0$$

with $p = (a, r, 1)^T$, $p' = (a', r', 1)^T$, $M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $M' = (R^T, -R^T 1)$

Essential Matrix (Longuet-Higgins, 1981)

$$p' \hat{e} p = 0 \quad \text{with} \quad \hat{e} = [e]_x R$$

Two ways to estimate the Essential Matrix

1. Calibration-based
2. Eight-Point Algorithm

Calibration-Based Method

1. From image of known calibration fixture, determine intrinsic parameters and extrinsic relation of two cameras.
2. Compute the relative position and orientation of the two cameras from $R_1, R_2, t_1, t_2$
3. Compute the Essential Matrix
The Eight-Point Algorithm (Longuet-Higgins, 1981)

For one point seen in two images

\[ p' \mathcal{E} p = 0 \quad \text{with} \quad \mathcal{E} = \{ E \}_{i} \mathbb{R} \]

\[
\begin{bmatrix}
E_{11} & E_{12} & E_{13} \\
E_{21} & E_{22} & E_{23} \\
E_{31} & E_{32} & E_{33}
\end{bmatrix}
\begin{bmatrix}
u' \\
v' \\
1
\end{bmatrix}
= 0
\]

For one point seen in two images, rearrange terms

\[
\begin{bmatrix}
u' \\
v' \\
1
\end{bmatrix}
= 0
\]

\[
\begin{bmatrix}
u' & u' & v' & u' & v' & u' & v' & E_{13}
\end{bmatrix}
= 0
\]

\[ \text{Solve as Eigenvector corresponding to the smallest Eigenvalue.} \]

Following, the text, view this as system of homogenous equations in \( E_{13} \) to \( E_{15} \)

- Set \( E_{13} \) to 1
- Use 8 points \((u,v)_i, i=1..8\)

\[
\begin{bmatrix}
u' & u' & v' & u' & v' & u' & v' & E_{13}
\end{bmatrix}
= 0
\]

Solve \( E_{13} \) to \( E_{15} \) --

The Essential Matrix

Example: converging cameras

Example: forward motion

Example: motion parallel with image plane

(simple for stereo -> rectification)
Properties of the Essential Matrix

\[ p' \mathbf{E} p = \mathbf{0} \quad \text{with} \quad \mathbf{E} = [t, |t| R] \]

- \( \mathbf{E} p' \) is the epipolar line associated with \( p' \).
- \( \mathbf{E}^T \mathbf{p} \) is the epipolar line associated with \( p \).
- \( \mathbf{E} \) is singular because \([t, |t| R]\) is singular and the product of two singular matrices is singular.
- \( \mathbf{E} e' = \mathbf{0} \) and \( \mathbf{E}^T e = \mathbf{0} \).
  Because \( \mathbf{E} \) is singular, one of the Eigenvalues is zero. The Eigenvector corresponding to the zero Eigenvalue is an epipole. The same holds for \( \mathbf{E}^T \).