CSE 203 Final Exam
Randomized Algorithms, Approximation, and Linear Programming Relaxation
Due June 16, 11:59 AM.

Answer both problems below. For each of the algorithm problems, design a polynomial-time algorithm with the best possible approximation ratio, or other standard as given in the problem. Your algorithm can be randomized or deterministic. Several problems have multiple solutions, with different ratios or other quality guarantees.

**Hitting multi-set** Say there are \( n \) types of jobs, each of which can be performed by up to \( k \) of the \( m \) types of machines. Which machines can perform which jobs is given as a bipartite graph. Since we will need to do similar jobs in parallel, we have a number \( r_j \) for each job; we will need to have at least \( r_j \) machines that can perform \( j \). We can buy multiple machines of the same type. Give a polynomial time algorithm with an approximation guarantee similar to those for the set cover problem.

**Bounding Sphere** Say that \( p_1, \ldots, p_n \) are points in \( \mathbb{R}^d \), the \( d \)-dimensional Euclidean space, i.e., each \( p_i \) is a vector of \( d \) real numbers. A bounding sphere \( S(q, R) \) is a sphere centered at point \( q \) (which is not necessarily one of the input points) of radius \( R \) that contains all of the \( p_i \), i.e., the distance between \( q \) and every \( p_i \) is at most \( R \). The Minimum Radius Bounding Sphere problem is, given such a set of points, find the bounding sphere with smallest radius \( R \). Give a polynomial time approximation scheme for this problem.