Photometric Stereo

Introduction to Computer Vision
CSE152
Lecture 16

Shading reveals 3-D surface geometry

Two shape-from-X methods that use shading

• Photometric stereo: Single viewpoint, multiple images under different lighting.
  1. Arbitrary known BRDF
  2. Lambertian BRDF, known lighting
  3. Lambertian BRDF, unknown lighting.

Announcements

• HW 3 due on Tuesday
• Any questions on stereo?

Two shape-from-X methods that use shading

• Shape-from-shading: Use just one image to recover shape. Requires knowledge of light source direction and BRDF everywhere. Too restrictive to be useful.

• Photometric stereo: Single viewpoint, multiple images under different lighting.
  1. Arbitrary known BRDF
  2. Lambertian BRDF, known lighting
  3. Lambertian BRDF, unknown lighting.

Photometric Stereo Rigs:
One viewpoint, changing lighting
An example of photometric stereo

Multi-view stereo vs. Photometric Stereo:
Assumptions
- Multi-view (binocular) Stereo
  - Multiple images
  - Dynamic scene
  - Multiple viewpoints
  - Fixed lighting
- Photometric Stereo
  - Multiple images
  - Static scene
  - Fixed viewpoint
  - Multiple lighting conditions

BRDF
- Bi-directional Reflectance Distribution Function
  \( \rho(\theta_{\text{in}}, \phi_{\text{in}}; \theta_{\text{out}}, \phi_{\text{out}}) \)
- Function of
  - Incoming light direction: \( \theta_{\text{in}}, \phi_{\text{in}} \)
  - Outgoing light direction: \( \theta_{\text{out}}, \phi_{\text{out}} \)
- Ratio of incident irradiance to emitted radiance

Photometric Stereo: Three problems
1. General but known reflectance function
2. Lambertian surfaces with known lighting
3. Lambertian surfaces with unknown lighting

Photometric Stereo:
General BRDF and Reflectance Map
Coordinate system

Surface: \( s(x,y) = (x,y,f(x,y)) \)
Tangent vectors:
\[
\frac{\partial s(x,y)}{\partial x} = (1,0, \frac{\partial f}{\partial x}) \\
\frac{\partial s(x,y)}{\partial y} = (0,1, \frac{\partial f}{\partial y})
\]

Normal vector
\[
\mathbf{n} = \begin{bmatrix}
\frac{\partial f}{\partial x} \\
\frac{\partial f}{\partial y} \\
-1
\end{bmatrix}
\]

Gradient Space (p,q)

Gradient Space: \((p,q)\)
\[
p = \frac{\partial f}{\partial x}, \quad q = \frac{\partial f}{\partial y}
\]

Reflectance Map
Let the BRDF be the same at all points on the surface, and let the light direction \( s \) be constant, and viewing direction constant.
1. Then image irradiance is a function of only the direction of the surface normal.
2. In gradient space, we can write \( E(p,q) \)
3. We can measure \( E(p,q) \) by taking an image of a sphere made of a single material under distant lighting

Example Reflectance Map: Lambertian surface
For lighting from front

Lambertian Reflectance Map
\[
\mathbf{E} = \mathbf{L} \rho \sqrt{1 + \rho_p^2 + \rho_q^2}
\]

Light Source Direction, expressed in gradient space
What does the intensity (irradiance) of one pixel in one image tell us?
It constrains the surface normal projecting to that point to a curve.

Two Light Sources
Two reflectance maps
A third image would disambiguate match.

Three Source Photometric stereo:
Step 1
Offline:
Using source directions & BRDF, construct reflectance map for each light source direction. \( R_1(p,q), R_2(p,q), R_3(p,q) \)
Online:
1. Acquire three images with known light source directions. \( E_1(x,y), E_2(x,y), E_3(x,y) \)
2. For each pixel location \((x,y)\), find \((p,q)\) as the intersection of the three curves
   \[ R_1(p,q) = E_1(x,y) \]
   \[ R_2(p,q) = E_2(x,y) \]
   \[ R_3(p,q) = E_3(x,y) \]
3. This is the surface normal at pixel \((x,y)\). Over image, the normal field is estimated.

An Example Normal Field

Plastic Baby Doll: Normal Field
Next step:
Go from normal field to surface

Recovering the surface \( f(x,y) \)
Many methods: Simplest approach
1. From normal field \( n = (n_x, n_y, n_z) \), \( p = -n_x/n_z \), \( q = -n_y/n_z \)
2. Integrate \( p = df/dx \) along a row \((x,0)\) to get \( f(x,0) \)
3. Then integrate \( q = df/dy \) along each column starting with value of the first row

What might go wrong?

- Height \( z(x,y) \) is obtained by integration along a curve from \((x_0, y_0)\):
- If one integrates the derivative field along any closed curve, one expects to get back to the starting value.
- Might not happen because of noisy estimates of \((p,q)\)

What might go wrong?
Integrability. If \( z(x,y) \) is the height function, we expect that:
In terms of estimated gradient space \((p,q)\), this means:
But since \( p \) and \( q \) were estimated independently at each point as intersection of curves on three reflectance maps, equality is not going to exactly hold

II. Photometric Stereo:
Lambertian Surface, Known Lighting

Lambertian Surface
At image location \((u,v)\), the intensity of a pixel \( x(u,v) \) is:
where
- \( \hat{a}(u,v) \) is the albedo of the surface projecting to \((u,v)\).
- \( \hat{n}(u,v) \) is the direction of the surface normal.
- \( s_0 \) is the light source intensity.
- \( s \) is the direction to the light source.
Lambertian Photometric stereo

- If the light sources $s_1$, $s_2$, and $s_3$ are known, then we can recover $b$ from as few as three images. (Photometric Stereo: Silver 80, Woodham 81)

$$[e_1 \ e_2 \ e_3] = \mathbf{b}^T [s_1 \ s_2 \ s_3]$$

- i.e., we measure $e_1$, $e_2$, and $e_3$, and we know $s_1$, $s_2$, and $s_3$. We can then solve for $\mathbf{b}$ by solving a linear system.

$$\mathbf{b}^T = \begin{bmatrix} e_1 & e_2 & e_3 \end{bmatrix} \begin{bmatrix} s_1 & s_2 & s_3 \end{bmatrix}^{-1}$$

- Normal is: $\mathbf{n} = \mathbf{b}/|\mathbf{b}|$, albedo is: $|\mathbf{b}|$

- Let $E_1$, $E_2$, $E_3$ be vectorized images, we can solve for all normals, scaled by the albedos as:

$$\mathbf{B} = \begin{bmatrix} E_1 & E_2 & E_3 \end{bmatrix} \begin{bmatrix} s_1 & s_2 & s_3 \end{bmatrix}^{-1}$$

What if we have more than 3 Images? Linear Least Squares

- Let the residual be $r = \mathbf{e} - \mathbf{Sb}$

- Rewrite as $\mathbf{e} = \mathbf{Sb}$

- Squaring this:

$$r^2 = (\mathbf{e} - \mathbf{Sb})^T (\mathbf{e} - \mathbf{Sb})$$

- where

$$\mathbf{e}$$ is $n$ by 1

$$\mathbf{b}$$ is 3 by 1

$$\mathbf{S}$$ is $n$ by 3

- $\nabla_{\mathbf{b}} (r^2) = 0$ - zero derivative is a necessary condition for a minimum, or

$$-2\mathbf{S}^T \mathbf{e} + 2\mathbf{S}^T \mathbf{Sb} = 0$$

- Solving for $\mathbf{b}$ gives

$$\mathbf{b} = (\mathbf{S}^T \mathbf{S})^{-1} \mathbf{S}^T \mathbf{e}$$
Lambertian Photometric Stereo

Reconstruction with albedo map

Without the albedo map

Another person

No Albedo map