1. Show that an affine transformation can map a circle to an ellipse, but cannot map an ellipse to a hyperbola or parabola.

2. Consider the Lyapunov map from MaSKS Equation (6.42), p. 195:

\[ L : \mathbb{C}^{3 \times 3} \rightarrow \mathbb{C}^{3 \times 3}, \quad X \mapsto X - CXC^\top \]

Assume \( C \) has \( n \) independent eigenvectors \( \{u_i \in \mathbb{C}^n\}_{i=1}^n \), with eigenvalues given by \( Cu_i = \lambda_i u_i \).

(a) Show that \( X_{ij} = u_i u_j^\top \in \mathbb{C}^{3 \times 3} \) is an eigenvector of \( L \). What is the corresponding eigenvalue?

(b) Assuming \( \det(C) = 1 \), which eigenvectors are in \( SRker(L) \)? In other words, for which values of \( i \) and \( j \) does \( L \) map the symmetric real \( X \) to the zero matrix?

(c) Explain the significance of \( SRker(L) \) if we interpret \( X \) as the coefficient matrix for a conic.

3. 2D Upgrade from Affine to Euclidean via Orthogonal Lines.

(a) Load in the affine-rectified image \( \text{affine_tile.gif} \), identify two pairs of imaged orthogonal lines, and plot them on the raw image.

(b) Implement the algorithm to solve for \( K \in SL(2)/SO(2) \) from two imaged right angles on a plane as described in H&Z Example 2.26 (Metric Rectification I), p. 56.

(c) Demonstrate your code on the tile image. Display the Euclidean-rectified image and plot the transformed line pairs on it.

4. Implement the algorithm described in H&Z Example 8.18 (A Simple Calibration Device), p. 211. Use it to estimate \( K \) from the image \( \text{squares.gif} \) depicting three metric planes. Compare your \( K \) to the \( K \) from H&Z.

5. Derive the solution for the homography \( H \in GL(4) \) relating \( n \geq 5 \) corresponding 3D points \((X_j^1, X_j^2), j = 1, 2, \ldots, n\), in general position.

6. Uncalibrated 3D Reconstruction.

(a) Run the script \( \text{house_views.m} \) to produce two uncalibrated views of a wireframe house.

(b) Recover \( F \) using the 8-pt. algorithm and compute the canonical camera matrices \((\Pi_1p, \Pi_2p)\).

(c) Triangulate to produce the projective structure \( X_p \).

(d) Find the homography \( H \in GL(4) \) to upgrade \( X_p \) directly to the Euclidean structure \( X_e \) using 5 ground truth points.

(e) Solve for the image of the plane at infinity \((v^\top, v_4)\) from three vanishing points and use it to upgrade \( X_p \) to the affine structure \( X_a \).