Homework #4

4.1. Five boys and three girls are throwing Frisbees. Each boy has one Frisbee and throws it to a random girl. What is the probability that each of the girls gets at least one of thrown Frisbees?

4.2. Consider the three permutations given in cycle form:
\[ f = (123)(45), \quad g = (12)(35)(4), \quad h = (1)(4352). \]
(a) What is \( f \circ g \) (expressed in cycle form)?
(b) What is \( g \circ h \circ f \)?
(c) What composition of \( f, g \) and \( h \) gives the permutation \( (1)(2)(345) \)?

4.3. A warped coin has probability of 0.5 of landing Heads, probability of 0.4 of landing Tails, and probability 0.1 of landing on its Edge. (I said it was warped!). It is flipped 5 times. What is the probability that more Heads occur than Tails?

4.4 6 cards are dealt (without replacement) from an ordinary deck of 52. Let \( X \) denote the number of Hearts that are dealt and let \( Y \) denote the number of face cards (= Jack, Queen or King) dealt. What is the expected value of \( X + Y \)?

4.5 A bin contains numbers the 10 numbers 1, 2, 3, . . . 10.
(a) 5 numbers are randomly drawn with replacement. What is the expected value of the sum of the numbers drawn?
(b) 5 numbers are randomly drawn without replacement. What is the expected value of the sum of the numbers drawn?

4.6 A biased coin has the probability of a Head of \( \frac{2}{3} \) and the probability of a Tail of \( \frac{1}{3} \). It is flipped 5 times. What is the expected number of times that a Head is followed immediately by a Tail?