CSE140 discussion section

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Administrivia

- Email: bryan at ucsd dot edu
- OH: Mon/Thur 2pm-3pm @ EBU3B B240A
- Discussion: Wed 2pm-3pm @ Center 109
- 9 homework, due before class
- 3 midterms, all in class (2/2, 2/23, 3/15)
- Optional take home final
cse140 and other cse courses

- cse20 discrete math
- cse140/L digital sys design
- cse141/L computer arch
- cse142 adv digital logic
- cse143 microelec sys design
- cse144 CAD / VLSI
- cse146 reliable hw
- cse148 adv proc design
What you want in a discussion section
What I was thinking of doing in discussion section

• Review of lecture materials from previous lectures
• Example problems
• Go over (graded) homework solutions
• Q&A

Post interesting problems on discussion board to be covered in discussion – help me help you
Topics covered yesterday

- Number representation
- How to add, sub, mult, div in binary
- Overflow detection
- Axiom and theorem in boolean algebra
Overflow detection methods

• Method 1
  – Overflow if the sign of sum is different from the sign of the operands
  – Caveat: There can be no overflow if the operand’s signs are different

• Method 2
  – Overflow if the carry-in and carry-out is different for the sign bit
More on overflow detection

<table>
<thead>
<tr>
<th>C_{n+1}</th>
<th>C_n</th>
<th>C_{n-1}</th>
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<tbody>
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<td>+</td>
<td>B_n</td>
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<tr>
<td>S_n</td>
<td>S_{n-1}</td>
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<table>
<thead>
<tr>
<th>Overflow?</th>
<th>Cin</th>
<th>A</th>
<th>B</th>
<th>S</th>
<th>Cout</th>
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• Method 1
  \[ F_1 (C_{in}, A, B) = A \cdot B \cdot \overline{S} + \overline{A} \cdot \overline{B} \cdot S = C_{in} \cdot A \cdot B + C_{in} \cdot \overline{A} \cdot \overline{B} \]

• Method 2
  \[ F_2 (C_{in}, C_{out}) = C_{in} \cdot C_{out} + C_{in} \cdot \overline{C_{out}} = C_{in} \oplus C_{out} \]

For fun: prove that \( F_1 = F_2 \)?
### Boolean function for S

<table>
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\[
S = \overline{C_{in}} \overline{A} B + \overline{C_{in}} A \overline{B} + C_{in} \overline{A} \overline{B} + C_{in} A B
\]

\[
= \overline{C_{in}} (\overline{A} B + A \overline{B}) + C_{in} (\overline{A} \overline{B} + A B)
\]

\[
= \overline{C_{in}} (\overline{A} B + A \overline{B}) + C_{in} (\overline{A} \overline{B} + A B + A \overline{A} + B \overline{B})
\]

\[
= \overline{C_{in}} (\overline{A} B + A \overline{B}) + C_{in} (A + \overline{B}) (\overline{A} + B)
\]

\[
= \overline{C_{in}} (\overline{A} B + A \overline{B}) + C_{in} (\overline{A} B) (A \overline{B})
\]

\[
= \overline{C_{in}} (\overline{A} B + A \overline{B}) + C_{in} (\overline{A} B + A \overline{B})
\]

\[
= C_{in} \overline{\varphi} (\overline{A} B + A \overline{B})
\]

\[
= C_{in} \overline{\varphi} (A \varphi B)
\]

- **distributive**
- **complement identity**
- **distributive**
- **deMorgan's**
- **deMorgan's**
- **XOR def**
- **XOR def**
Boolean function for Cout

<table>
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Cout = \overline{C_{in}} A B + C_{in} \overline{A} B + C_{in} A \overline{B} + C_{in} A B

Cout = \overline{C_{in}} A B + C_{in} A B
+ C_{in} \overline{A} B + C_{in} A B
+ C_{in} A \overline{B} + C_{in} A B

Cout = A B (C_{in} + \overline{C_{in}})
+ C_{in} B (A + \overline{A})
+ C_{in} A (B + \overline{B})

Cout = A B + C_{in} A + C_{in} B

idempotency
distributive
complement
identity
Is Method 1 == Method 2?

\[
F_2 (C_{in}, C_{out}) = C_{in} C_{out} + C_{in} \overline{C_{out}}
\]

\[
F_2 (C_{in}, A, B) = C_{in} (A B + B C_{in} + C_{in} A) + C_{in} (A B + B C_{in} + C_{in} A)
\]

\[
F_2 (C_{in}, A, B) = C_{in} A B + C_{in} (A B + B C_{in} + C_{in} A)
\]

\[
F_2 (C_{in}, A, B) = C_{in} A B + C_{in} (A B) (B + C_{in}) (C_{in} + A)
\]

\[
F_2 (C_{in}, A, B) = C_{in} A B + C_{in} (A + B) (B + C_{in}) (C_{in} + A)
\]

\[
F_2 (C_{in}, A, B) = C_{in} A B + C_{in} (A + B) B A
\]

\[
F_2 (C_{in}, A, B) = C_{in} A B + C_{in} A \overline{B}
\]

Yup, method 2 is equal to method 1.