Problem 1 Let $COMPL_{DFA}$ be the language

$$\{ \langle A, B \rangle \mid A \text{ and } B \text{ are DFAs over the same alphabet } \Sigma \text{ and } L(A) = \overline{L(B)} \} .$$

(Notice the complementation bar over $L(B)$ above!) Show that $COMPL_{DFA}$ is decidable.

Problem 2 We say that a string over the alphabet $\Sigma = \{0, 1\}$ is sorted if any 0s in it occur before any 1s. (For example, 111 is sorted, whereas 00110 is not.) We consider the empty string to be sorted. Let $MESSY_{DFA}$ be the language

$$\{ \langle A \rangle \mid A \text{ is a DFA over the alphabet } \Sigma = \{0, 1\} \text{ and no string in } L(A) \text{ is sorted} \} .$$

Show that $MESSY_{DFA}$ is decidable.

**Hint:** Think about intersecting two regular languages.

Problem 3 (Sipser 4.22) A useless state in a pushdown automaton is never entered on any input string. Consider the problem of determining whether a pushdown automaton has any useless states. Formulate this problem as a language and show that it is decidable.

Problem 4 $NOTSTATE$ be the language

$$\{ \langle M, w, q \rangle \mid M \text{ is a Turing machine, } w \text{ is a string, and } q \text{ is a state;} \text{ and } M, \text{ when run on input } w, \text{ never enters the state } q. \} .$$

Show that $NOTSTATE$ is undecidable.

**Hint:** Assume that $NOTSTATE$ is decidable, and use a decider for $NOTSTATE$ to decide the acceptance problem $A_{TM}$, yielding a contradiction.