CSE20 Exercise 1, February 12, 2010,

1. Residual Number System: Represent 38 with a residual number system of moduli (m_1, m_2, m_3)= (3, 5, 7).
2. Residual Number System: Suppose $(x\%5, x\%7, x\%11) = (1, 2, 3)$, where symbol % denotes modulus operation. Find the smallest positive integer x that satisfies this system.
3. Residual Number System: Show the operation of $38 + 44$ in a residual number system with moduli $(m_1, m_2, m_3) = (3, 5, 7)$.
4. Residual Number System: Show the operation of 19×15 in a residual number system with moduli $(m_1, m_2, m_3) = (5, 13, 14)$.
5. Residual Number System: State and prove the Chinese remainder theorm.
6. Boolean Algebra: Prove that for any a and b in the set B of a Boolean algebra, $(a + b)(a + b') = a$.
7. Boolean Algebra: Prove general associativity holds for + in any Boolean algebra. For all $n \geq 1$,

$$a_1 + (a_2 + (a_2 + (\ldots + a_n))) = (((a_1 + a_2) + a_3) + \ldots) + a_n$$

You may assume that associativity holds for $n = 2$.
8. Boolean Algebra: State and prove DeMorgan's laws.
9. Boolean Algebra: Show the operation tables for a Boolean algebra of four elements.
10. Boolean Algebra: Simplify formula $(pq + r')(p + r)(q + r)$.
11. Boolean Algebra: Express Boolean function $E(x, y, z) = (x' + y)(xy)'(x + y' + z)$ in sum-of-products form.
12. Boolean Algebra: Express Boolean function $E(x, y, z) = xy + (x + z)' + x'y'z$ in product-of-sums form.
13. Boolean Algebra: Prove or disprove the Boolean equation, $(a'b' + c)(a + b)(b' + ac)' = a'bc$.
14. Boolean Algebra: Reduce the following to an expression of a minimal number of literals (4). $abc'd + ab'c + bc'd + ab'c' + acd + a'bcd$.