

# Solution of CSE20 Exercise 1

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1.  $(1000110100)_2$

2. 53

3.  $3^5 \times 4^4$ , or  $0 \sim (3^5 \times 4^4 - 1)$

4.

$x$	$y$	$b_{in}$	$b_{out}$	$d$
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	1	0
1	0	0	0	1
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

5.

id	b2	b1	b0
0	0	0	0
1	0	0	1
2	0	1	1
3	0	1	0
4	1	1	0
5	1	1	1
6	1	0	1
7	1	0	0

6. Signed magnitude & one's complement:  $-127 \sim 127$   
Two's complement:  $-128 \sim 127$

7.  $(01000010)_2$

8.  $x = 12 = 1100_2$   
 $y = 14 = 1110_2 = 001110_2$   
 By 6-bit one's complement,  $x - y = x + (-y) = 001100_2 + 110001_2 = 111101_2$

9. Define eight's complement so that  $-x$  is represented as n-digit  $8^n - x$ .  
 In 5-digit system,  $-y = 8^5 - 15_8 = 77763_8$ ,  
 $x - y = x + (-y) = 00011_8 + 77763_8 = 77774_8$  (which equals  $-4$ )

10. (a) Proof without induction:

$$\begin{aligned} & \frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \cdots + \frac{1}{(2n-1)(2n+1)} \\ &= \frac{1}{2} \left( \frac{1}{1} - \frac{1}{3} \right) + \frac{1}{2} \left( \frac{1}{3} - \frac{1}{5} \right) + \cdots + \frac{1}{2} \left( \frac{1}{2n-1} - \frac{1}{2n+1} \right) \\ &= \frac{1}{2} \left( \frac{1}{1} \right) + \frac{1}{2} \left( -\frac{1}{3} + \frac{1}{3} \right) + \cdots + \frac{1}{2} \left( -\frac{1}{2n-1} + \frac{1}{2n-1} \right) + \frac{1}{2} \left( -\frac{1}{2n+1} \right) \\ &= \frac{1}{2} \left( 1 - \frac{1}{2n+1} \right) \\ &= \frac{n}{2n+1} \end{aligned}$$

- (b) Proof with induction:

When  $n = 1$ ,  $\frac{1}{1 \times 3} = \frac{1}{2+1}$ , the equation is true.

Assume for  $n = k$ ,  $\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \cdots + \frac{1}{(2k-1)(2k+1)} = \frac{k}{2k+1}$  is true,  
 so when  $n = k + 1$ , using our assumption on  $n = k$ ,

$$\begin{aligned} & \frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \cdots + \frac{1}{(2k-1)(2k+1)} + \frac{1}{(2k+1)(2k+3)} \\ &= \frac{k}{2k+1} + \frac{1}{(2k+1)(2k+3)} \\ &= \frac{2k^2 + 3k + 1}{(2k+1)(2k+3)} \\ &= \frac{(2k+1)(k+1)}{(2k+1)(2k+3)} \\ &= \frac{k+1}{2k+3} \end{aligned}$$

the equation is also true. Induction complete.

11. Proof: Let  $x = q_x d + r_x$ ,  $y = q_y d + r_y$ ,  
 $(x + y) \% d = (q_x d + r_x + q_y d + r_y) \% d = (r_x + r_y) \% d = (x \% d + y \% d) \% d$

12. Proof: Let  $x = q_x d + r_x$ ,  $y = q_y d + r_y$ ,  
 $(x - y) \% d = [(q_x d + r_x) - (q_y d + r_y)] \% d = (r_x - r_y) \% d = (x \% d - y \% d) \% d$

13. Proof: Let  $x = q_x d + r_x$ ,  $y = q_y d + r_y$ ,  
 $(x \times y) \% d$   
 $= [(q_x d + r_x) \times (q_y d + r_y)] \% d$   
 $= (q_x q_y d^2 + q_x r_y d + q_y r_x d + r_x r_y) \% d$   
 $= (r_x r_y) \% d$   
 $= (x \% d \times y \% d) \% d$