

# CSE20

## Lecture 2: Number Systems: Binary Numbers, Gray Code, and Negative Numbers

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# Number Systems

1. Introduction
2. Binary Numbers
3. Gray code
4. Negative Numbers
5. Residual Numbers



# 2. Binary Cont.

a	b	Carry	Sum
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

**RULE:**

$$2 \times \text{Carry} + \text{Sum} = a + b + c$$

id	a	b	c	Carry	Sum
0	0	0	0	0	0
1	0	0	1	0	1
2	0	1	0	0	1
3	0	1	1	1	0
4	1	0	0	0	1
5	1	0	1	1	0
6	1	1	0	1	0
7	1	1	1	1	1

$$2 \times 0 + 0 = 0 \quad 0 \quad 0 \quad \text{id 0}$$

$$2 \times 0 + 1 = 0 \quad 0 \quad 1 \quad \text{id 1}$$

$$2 \times 1 + 0 = 1 \quad 1 \quad 0 \quad \text{id 6}$$

$$2 \times 1 + 1 = 1 \quad 1 \quad 1 \quad \text{id 7}$$

# 3. Gray Code

ld	b3	b2	b1	b0
0	0	0	0	0
1	0	0	0	1
2	0	0	1	1
3	0	0	1	0
4	0	1	1	0
5	0	1	1	1
6	0	1	0	1
7	0	1	0	0
8	1	1	0	0
9	1	1	0	1
10	1	1	1	1
11	1	1	1	0
12	1	0	1	0
13	1	0	1	1
14	1	0	0	1
15	1	0	0	0

} reflection

Low power (reliability) when the numbers are consecutive in series.

The idea is to only change ONE bit at a time.

e.g. addresses, analog signals

**NOTE: Not for arithmetic operations (the rule is too complicated)**

# 4. Negative Numbers

**Given a positive integer  $x$ , represent the negative integer  $-x$  in  $(b_{n-1}, \dots, b_0)$**

**(i) Signed bit system**

**$b_{n-1}=1$ : negative,  $(b_{n-2}, \dots, b_0)=x$ .**

**(ii) One's Complement**

**Present  $2^n - 1 - x$  in binary.**

**(iii) Two's Complement**

**Present  $2^n - x$  in binary. Ignore bit  $b_n$ .**

# 4. Negative Numbers

NOTE: Back to binary system

## Deriving One's and Two's

## Reverse Derivation

(i) Signed bit - x  
b3: negative

(ii) One's Complement  
 $2^n - 1 - x$

(iii) Two's Complement  
 $2^n - x$

n is the number of bits (in this case n=4)

Use the above formulas to solve for x when number is negative

One's Complement      Two's Complement

$$8 = 16 - 1 - x$$

$$8 = 16 - x$$

$$9 = 16 - 1 - x$$

id	b <sub>3</sub>	b <sub>2</sub>	b <sub>1</sub>	b <sub>0</sub>	Signed	One's	Two's	Two's	(b <sub>4</sub> )	b <sub>3</sub>	b <sub>2</sub>	b <sub>1</sub>	b <sub>0</sub>
0	0	0	0	0	0	0	0	7	1	0	1	1	1
1	0	0	0	1	1	1	1	6	1	0	1	1	0
2	0	0	1	0	2	2	2	5	1	0	1	0	1
3	0	0	1	1	3	3	3	4	1	0	1	0	0
4	0	1	0	0	4	4	4	3	1	0	0	1	1
5	0	1	0	1	5	5	5	2	1	0	0	1	0
6	0	1	1	0	6	6	6	1	1	0	0	0	1
7	0	1	1	1	7	7	7	0	1	0	0	0	0
8	1	0	0	0	-0	-7	-8	-1	0	1	1	1	1
9	1	0	0	1	-1	-6	-7	-2	0	1	1	1	0
10	1	0	1	0	-2	-5	-6	-3	0	1	1	0	1
11	1	0	1	1	-3	-4	-5	-4	0	1	1	0	0
12	1	1	0	0	-4	-3	-4	-5	0	1	0	1	1
13	1	1	0	1	-5	-2	-3	-6	0	1	0	1	0
14	1	1	1	0	-6	-1	-2	-7	0	1	0	0	1
15	1	1	1	1	-7	-0	-1	-8	0	1	0	0	0