CSE 20 Lecture 4
01/14/10
CK Cheng, UC San Diego

- Negative numbers
  One’s and Two’s Complement
- Residual numbers
Announcements

• CK No office hours next week
• Pat No office hours next week
• On Tuesday 1/19, Peng Du will be going over Chapter 11 (Shaum’s)
• On Thursday 1/21, Peng Du will be going over Chapter 11 (Shaum’s)
• Midterm will be held on 1/28
One’s Complement

• Given two positive integers \( x \) & \( y \), perform:
  • \( x + y \)
  • \( x - y \)
  • \( -x + y \)
  • \( -x - y \)
How to Solve

1) Derive 1’s complement of the operands
2) Sum up the two operands
3) Use carry-out to feed into carry-in
4) The result is the solution in 1’s complement
<table>
<thead>
<tr>
<th>Arithmetic</th>
<th>1’s Complement</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x+y$</td>
<td>$x+y$</td>
</tr>
<tr>
<td>$x-y$</td>
<td>$x+(2^n-1-y)=2^n-1+(x-y)$</td>
</tr>
<tr>
<td>$-x+y$</td>
<td>$(2^n-1-x)+y = 2^n-1+(-x+y)$</td>
</tr>
<tr>
<td>$-x-y$</td>
<td>$(2^n-1-x)+(2^n-1-y)=2^n-1+(2^n-1-x-y)$</td>
</tr>
</tbody>
</table>
1’s Complement

Example: $4 - 3 = 1$

3 in binary = 0011. Flipping the bits, you get −3 (1100) in binary, which is 12.

\[
\begin{array}{c c c}
0100 & \text{(4 in binary)} \\
+1100 & \text{(12 in binary)} \\
10000 & =16 \text{ (15+1)} \\
\end{array}
\]

So now take the extra 1 and remove it from the 5th spot and add it to the remainder

\[
\begin{array}{c c c}
0000 \\
+1 & \\
0001 & \text{(1 in 1’s comp)}
\end{array}
\]
1’s Complement

Example: \(-4 + 3 = -1\)

4 in binary = 0100. Flipping the bits, you get \(-4\) (1011) in binary, which is 11.

\[
\begin{array}{c}
1011 & \text{ (11 in binary) } \\
+0011 & \text{ (3 in binary) } \\
1110 & \text{ (=14 in binary (15-1), but in 1’s comp, is -1) }
\end{array}
\]
1’s Complement

Example: $-4 - 3 = -7$

4 in binary = 0100. Flipping the bits, you get $-4$ (1011) in binary, which =11.
3 in binary = 0011. Flipping the bits, you get $-3$ (1100) in binary, which =12.

\[
\begin{align*}
1011 & \quad (11 \text{ in binary, or } 15-4) \\
+1100 & \quad (12 \text{ in binary, or } 15-3) \\
10111 & \quad (=23 \text{ in binary } (15+15-7))
\end{align*}
\]

So now take the extra 1 and remove it from the 5\textsuperscript{th} spot and add it to the remainder

\[
\begin{align*}
0111 \\
+1 & \\
1000 & \quad (-7 \text{ in 1’s comp})
\end{align*}
\]
Recovery of the Numbers

1's Compliment

Let \( f(x) = 2^n - 1 - x \)

Theorem: \( f(f(x)) = x \)

Proof: \( f(f(x)) \)
    \[ = f(2^n - 1 - x) \]
    \[ = 2^n - 1 - (2^n - 1 - x) \]
    \[ = x \]

2's Compliment

Let \( g(x) = 2^n - x \)

Theorem: \( g(g(x)) = x \)

Proof: \( g(g(x)) \)
    \[ = g(2^n - x) \]
    \[ = 2^n - (2^n - x) \]
    \[ = x \]
Residual Numbers
(NT-1, Chp 11 Shaums)

1) Introduction

2) Definitions

3) Operations
Residual Numbers - Introduction

Goal: Simplify arithmetics (+ – x) when bit width n is huge, eg. n=1000.

Definition: Mod (Modulus) operation

\[ \text{Integer } x = q \times d + r, \ 0 \leq r < d, \]

where q : quotient, d : divisor, and r : remainder.
Mod: $x \% d = r$

**Examples:**

- $0 \% 3 = 0$
- $2 \% 3 = 2$
- $21 \% 3 = 0$
- $0 \% 5 = 0$
- $2 \% 5 = 2$
- $21 \% 5 = 1$
- $0 \% 7 = 0$
- $2 \% 7 = 2$
- $21 \% 7 = 0$

**Negative Numbers:**

- $-3 \% 3 = 0$
- $-3 \% 5 = 2$  
  to solve: $-3 = q \times 5 + r$
  
  $q$ has to be equal to $-1$ as the remainder can only be positive

- $-3 \% 7 = 4$  
  to solve: $-3 = q \times 7 + r$
  
  $q$ has to be equal to $-1$ as the remainder can only be positive

- $-21 \% 3 = 0$

- $21 \% 5 = 4$  
  to solve: $-21 = q \times 5 + r$
  
  $q = -5$, $r = 4$ as $-21 = -25 + 4$