Image Formation and Cameras (cont.)
Biometrics
CSE 190A
Lecture 6

Announcements

- Project topics due today

Image Formation: Outline
- Factors in producing images
- Projection
- Perspective
- Vanishing points
- Orthographic
- Lenses
- Sensors
- Quantization/Resolution
- Illumination
- Reflectance

Pinhole Camera: **Perspective projection**
- Abstract camera model - box with a small hole in it

Homogenous coordinates
- Our usual coordinate system is called a Euclidean or affine coordinate system
- Rotations, translations and projection in Homogenous coordinates can be expressed linearly as matrix multiplies

Euclidean -> Homogenous-> Euclidean

In 2-D
- Euclidean -> Homogenous: \((x, y) \rightarrow k (x,y,1)\)
- Homogenous -> Euclidean: \((u,v,w) \rightarrow (u/w, v/w)\)

In 3-D
- Euclidean -> Homogenous: \((x, y, z) \rightarrow k (x,y,z,1)\)
- Homogenous -> Euclidean: \((x, y, z, w) \rightarrow (x/w, y/w, z/w)\)
The equation of projection

Cartesian coordinates:

\[(x, y, z) \rightarrow \left( \frac{x}{z}, \frac{y}{z} \right)\]

Homogenous Coordinates and Camera matrix

\[
\begin{bmatrix}
U' \\
V' \\
W'
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 / f & 0
\end{bmatrix}
\begin{bmatrix}
X' \\
Y' \\
Z' \\
1
\end{bmatrix}
\]

Affine Camera Model

- Take Perspective projection equation, and perform Taylor Series Expansion about \((x_0, y_0, z_0)\).
- Drop terms of higher order than linear.
- Resulting expression is affine camera model

\[
\begin{bmatrix}
u \\
v
\end{bmatrix} =
\begin{bmatrix}
1 / x_0 & 0 & -x_0 / z_0^2 \\
0 & 1 / z_0 & -y_0 / z_0^2
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} = A p + b
\]

Rewrite Affine camera model in terms of Homogenous Coordinates

\[
\begin{bmatrix}
u \\
v \\
w
\end{bmatrix} =
\begin{bmatrix}
1 / z_0 & 0 & -x_0 / z_0^2 & x_0 \\
0 & 1 / z_0 & -y_0 / z_0^2 & y_0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
\]

Scaled Orthographic projection

Starting with Affine camera mode

Take Taylor series about \((0, 0, z_0)\) – a point on optical axis

\[
\begin{bmatrix}
u \\
v
\end{bmatrix} =
\begin{bmatrix}
1 / z_0 & 0 & 0 & 0 \\
0 & 1 / z_0 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
\]
What if camera coordinate system differs from object coordinate system

\[ \text{Coordinate Changes: Rigid Transformations} \]

\[ B\mathbf{P} = B\mathbf{R}\mathbf{A}\mathbf{P} + B\mathbf{O}_A \]

Rotation Matrix

Translation vector

A rotation matrix \( \mathbf{R} \) has the following properties:

\- Its inverse is equal to its transpose \( \mathbf{R}^{-1} = \mathbf{R}^T \)
\- Its determinant is equal to 1: \( \det(\mathbf{R}) = 1 \)

Or equivalently:

\- Rows (or columns) of \( \mathbf{R} \) form a right-handed orthonormal coordinate system.

Rotation: Homogenous Coordinates

\[ \begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \]

Roll-Pitch-Yaw

\[ R = \text{rot}(\hat{i}, \alpha)\text{rot}(\hat{j}, \beta)\text{rot}(\hat{k}, \gamma) \]

Euler Angles

\[ R = \text{rot}(\hat{k}', \alpha)\text{rot}(\hat{j}', \beta)\text{rot}(\hat{k}, \gamma) \]

Rotation

\[ \begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} kk(1-c)+c & kk(1-c)k\beta & kk(1-c)+c & 0 \\ kk(1-c)+k\beta & kk(1-c)+c & kk(1-c)-k\beta & 0 \\ kk(1-c)-k\beta & kk(1-c)+k\beta & kk(1-c)+c & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \]

where \( c = \cos \theta \) and \( s = \sin \theta \)
Camera parameters

- **Issue**
  - camera may not be at the origin, looking down the z-axis
  - extrinsic parameters (Rigid Transformation)
  - one unit in camera coordinates may not be the same as one unit in world coordinates
  - intrinsic parameters - focal length, principal point, aspect ratio, angle between axes, etc.

\[
\begin{bmatrix}
X \\
Y \\
Z \\
1
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Camera Calibration

Given n points \( P_1, \ldots, P_n \) with known positions and their images \( p_1, \ldots, p_n \), estimate intrinsic and extrinsic camera parameters

- See Text book for how to do it.

Camera Obscure

"When images of illuminated objects ... penetrate through a small hole into a very dark room ... you will see [on the opposite wall] these objects in their proper form and color, reduced in size ... in a reversed position, owing to the intersection of the rays".

Da Vinci

http://www.acmi.net.au/AIC/CAMERA_OBSCURA.html (Russell Naughton)

Getting more light – Bigger Aperture

Pinhole Camera Images with Variable Aperture

- 2 mm
- 1 mm
- 0.6 mm
- 0.35 mm
- 0.15 mm
- 0.07 mm

The reason for lenses
A price: Whereas the image of $P$ is in focus, the image of $Q$ isn’t.

Deviations from the lens model

Deviations from this ideal are **aberrations**

**Two types**

1. **geometrical**
   - spherical aberration
   - astigmatism
   - distortion
   - coma

2. **chromatic**

Aberrations are reduced by combining lenses

Spherical aberration

rays parallel to the axis do not converge

outer portions of the lens yield smaller focal lengths
**Distortion**
magnification/focal length different for different angles of inclination

- **Pincushion** (tele-photo)
- **Barrel** (wide-angle)

Can be corrected! (if parameters are known)

**Chromatic Aberration**
Index of refraction of lens depends on wavelength of light

**Chromatic Aberration**
- Rays of different wavelengths focus in different planes
- The image is blurred and appears colored at the fringe.

Sometimes **achromatization** is achieved for more than 2 wavelengths

**Vignetting: Spatial Non-Uniformity**
- Only part of the light reaches the sensor
- Periphery of the image is dimmer

**Image Brightness**
How Cameras Produce Images

- Basic process:
  - Photons hit a detector
  - The detector becomes charged
  - The charge is read out as brightness

- Sensor types:
  - CCD (charge-coupled device)
    - High sensitivity
    - High power
    - Cannot be individually addressed
    - Blooming
  - CMOS
    - Most common
    - Simple to fabricate (cheap)
    - Lower sensitivity, lower power
    - Can be individually addressed

Camera’s sensor

- Measured pixel intensity is a function of irradiance integrated over
  - Pixel’s area
  - Over a range of wavelengths
  - For some time

\[ I = \iiint E(x, y, \lambda, t) s(x, y) q(\lambda) dy dx dt \]

Light at surfaces

Many effects when light strikes a surface could be:
- Transmitted
  - Skin, glass
- Reflected
  - Mirror
  - Scattered
  - Milk
- Travel along the surface and leave at some other point
- Absorbed
  - Sweaty skin

Assume that
- Surfaces don’t fluoresce
  - E.g. Scorpions, detergents
- Surfaces don’t emit light
  (i.e. are cool)
- All the light leaving a point is due to that arriving at that point

BRDF

- Bi-directional Reflectance Distribution Function
  \[ \rho(\theta_{in}, \phi_{in}; \theta_{out}, \phi_{out}) \]
- Function of
  - Incoming light direction: \( \theta_{in}, \phi_{in} \)
  - Outgoing light direction: \( \theta_{out}, \phi_{out} \)
- Ratio of incident irradiance to emitted radiance

Surface Reflectance Models

Common Models
- Lambertian
- Phong
- Physics-based
  - Specular
    [Hinn 1977], [Cook-Torrance 1982], [Ward 1992]
  - Diffuse
    [Hamm, Kreuger 1993]
  - Generalized Lambertian
    [Oren, Nayar 1995]
  - Thoroughly Pitted Surfaces
    [Koenderink et al. 1999]
- Phenomenological
  [Koenderink, Van Doom 1996]

Arbitrary Reflectance
- Non-parametric model
- Anisotropic
- Non-uniform over surface
- BRDF Measurement
  [Dana et al. 1999], [Marchchner]

Lambertian Surface

At image location \((u, v)\), the intensity of a pixel \(x(u, v)\) is:

\[ x(u, v) = \left[a(u, v) \cdot \hat{n}(u, v) \cdot s \right] \cdot b(u, v) \cdot s \]

where
- \(a(u, v)\) is the albedo of the surface projecting to \((u, v)\).
- \(\hat{n}(u, v)\) is the direction of the surface normal.
- \(s\) is the light source intensity.
- \(\hat{n}\) is the direction to the light source.
Specular Reflection: Smooth Surface

Phong – rough, specular

Rough Specular Surface

Symmetric V-shaped grooves – ‘microfacets’

Phong Lobe

Phong Model

Mirror  Diffuse

Gloss removal