Quiz 1 — CSE 20, Winter 2009

PRINT your name here: ______________________

- Print your name **immediately** on the cover page, as well as **each page** of the exam, in the space provided. Each time you are caught working on a page without your name printed on it, you will lose one point.

- This exam is closed book. You are only allowed to use one page of notes (double sided is fine)

- Your solution will be evaluated both for correctness and clarity. A poorly written solution won’t get full credit even if correct.

- Read all the problems first before start working on any of them, so you can manage your time wisely

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>96</strong></td>
<td></td>
</tr>
</tbody>
</table>
1 Reading Propositional Logic [6 points]

Let P, Q and R be the propositions

- P: You have the flu
- Q: You miss the final examination
- R: You get an A in the course

Express each of the following propositions as an English sentence:

1. $P \rightarrow Q$
   If you have the flu, then you miss the final.

2. $Q \rightarrow \neg R$
   If you miss the final, you don’t get an A in the course.

3. $\neg Q \leftrightarrow R$
   You get an A in the course if and only if you don’t miss the final examination.
2 Negation [8 points]

Two of the propositions below are one the negation of the other. Determine which ones, and prove your answer correct using the truth table method.

1. \( P \land Q \land R \)

2. \((P \to \neg R) \lor (Q \to \neg R)\)

3. \((P \land Q) \lor (\neg Q \land R)\)

<table>
<thead>
<tr>
<th>(P)</th>
<th>(Q)</th>
<th>(R)</th>
<th>(P \land Q \land R)</th>
<th>(P \to \neg R)</th>
<th>(Q \to \neg R)</th>
<th>((P \to \neg R) \lor (Q \to \neg R))</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

From the above truth table we can see 1 and 2 are negation of each other.
3 Writing Propositional Logic [6 points]

Let $K$, $L$ and $M$ be the propositions

- $K$: You get an A on the final exam
- $L$: You do every exercise in the book
- $M$: You get an A in the class

Write the following propositions using $K$, $L$, $M$ and the logical connectives $\land$, $\lor$, $\rightarrow$, $\leftrightarrow$, $\neg$.

1. You get an A in the final, you do every exercise in the book, and you get an A in the class
   
   $K \land L \land M$

2. You will get an A in this class if and only if you either do every exercise in the book, or you get an A on the final.
   
   $M \leftrightarrow (K \lor L)$

3. To get an A in the class, you must get an A on the final
   
   $M \rightarrow K$
4 Equivalence [11 points]

For each of the following equivalences, mark if they are true or false. [5 points]

1. T ✔  $P \land (P \lor Q) \equiv Q$
2. T ✔  $(P \rightarrow Q) \rightarrow R \equiv P \rightarrow (Q \rightarrow R)$
3. T ✔  $\neg(P \leftrightarrow Q) \equiv (\neg P) \leftrightarrow (\neg Q)$
4. ✔ F  $P \lor (P \land Q) \equiv P$
5. ✔ F  $\neg(P \leftrightarrow Q) \equiv (\neg P) \leftrightarrow Q$

[Optional] The following list contains 3 pairs of equivalent propositions. Find the 3 matching pairs. [6 points]

1. $(P \leftrightarrow Q) \rightarrow R$
2. $R \rightarrow (P \rightarrow Q)$
3. $(P \land \neg Q) \lor (\neg P \land Q) \lor R$
4. $\neg R \lor \neg P \lor Q$
5. $(P \rightarrow R) \leftrightarrow Q$
6. $((\neg P \lor R) \land Q) \lor (\neg Q \land P \land \neg R)$

(1, 3), (2, 4), and (5, 6)
5 Logic Circuits [5 points]

Draw a circuit (using AND, OR and NOT gates) equivalent to the expression
\((A \lor \neg B) \land (B \lor \neg C)\).

6 NAND Circuit [5 points]

Draw a circuit (using only NAND gates) equivalent to the expression \(A \rightarrow B\),
and the corresponding formula using the Sheffer stroke |.

\[ A \rightarrow B \equiv A|(B|B) \equiv A|(A|B) \]
7 Synthesis [8 points]

Give a propositional statement (in the variables $A, B, C$, and using any connective $\land, \lor, \neg, \rightarrow, \leftrightarrow$ you may find useful) corresponding to the following truth table. For full credit, give a shorter formula.

<table>
<thead>
<tr>
<th>$A$</th>
<th>$B$</th>
<th>$C$</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>$T$</td>
<td>$T$</td>
<td>$T$</td>
</tr>
<tr>
<td>$T$</td>
<td>$T$</td>
<td>$F$</td>
<td>$T$</td>
</tr>
<tr>
<td>$T$</td>
<td>$F$</td>
<td>$T$</td>
<td>$F$</td>
</tr>
<tr>
<td>$T$</td>
<td>$F$</td>
<td>$F$</td>
<td>$T$</td>
</tr>
<tr>
<td>$F$</td>
<td>$T$</td>
<td>$T$</td>
<td>$T$</td>
</tr>
<tr>
<td>$F$</td>
<td>$T$</td>
<td>$F$</td>
<td>$F$</td>
</tr>
<tr>
<td>$F$</td>
<td>$F$</td>
<td>$T$</td>
<td>$T$</td>
</tr>
<tr>
<td>$F$</td>
<td>$F$</td>
<td>$F$</td>
<td>$F$</td>
</tr>
</tbody>
</table>

There are many different solution:

1. $(A \land (C \rightarrow B)) \lor (\neg A \land C)$
2. $(A \land \neg C) \lor (A \land B) \lor (\neg A \land C)$
3. $(A \land \neg B \land \neg C) \lor (A \land B) \lor (\neg A \land C)$
4. $(A \land \neg C) \lor (C \land (B \lor \neg A))$
5. $(A \lor C) \land ((A \land C) \rightarrow B)$
8 Predicates [5 points]

For each of the following statements, state if the statement is valid or not. In all examples, the variables $x, y$ range over the set $R$ of all real numbers.

1. $\exists x. x^2 = x$  
   - $\Box$  
   - $\bigcirc$ 

2. $\forall x. \exists y. x = y^2$  
   - $T$  
   - $\bigcirc$ 

3. $\forall x. \exists y. x^2 = y$  
   - $\Box$  
   - $\bigcirc$ 

4. $\forall x. \exists y. (y > x) \rightarrow (x^2 = x)$  
   - $\Box$  
   - $\bigcirc$ 

5. $\exists x. \forall y. x = y + 1$  
   - $T$  
   - $\bigcirc$ 

9 Deductions [8 points]

For each of the following pairs of predicates $A(x)$ and $B(x)$, determine if $A(x) \Rightarrow B(x)$ and/or $B(x) \Rightarrow A(x)$ (for any meaning of the predicates $P(x)$ and $Q(x)$) and mark the corresponding box with T or F.

<table>
<thead>
<tr>
<th>$A(x)$</th>
<th>$B(x)$</th>
<th>$A(x) \Rightarrow B(x)$</th>
<th>$A(x) \Leftarrow B(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\forall x. P(x)) \land (\forall x. Q(x))$</td>
<td>$\forall x. (P(x) \land Q(x))$</td>
<td>$T$</td>
<td>$T$</td>
</tr>
<tr>
<td>$(\forall x. P(x)) \rightarrow (\forall x. Q(x))$</td>
<td>$\forall x. (P(x) \rightarrow Q(x))$</td>
<td>$F$</td>
<td>$T$</td>
</tr>
<tr>
<td>$(\exists x. P(x)) \lor (\exists x. Q(x))$</td>
<td>$\exists x. (P(x) \lor Q(x))$</td>
<td>$T$</td>
<td>$T$</td>
</tr>
<tr>
<td>$(\exists x. P(x)) \land (\exists x. Q(x))$</td>
<td>$\exists x. (P(x) \land Q(x))$</td>
<td>$F$</td>
<td>$T$</td>
</tr>
</tbody>
</table>
A discrete math class has 33 students distributed as follows

<table>
<thead>
<tr>
<th></th>
<th>math major</th>
<th>cs major</th>
</tr>
</thead>
<tbody>
<tr>
<td>freshmen</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>sophomores</td>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>junior</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>senior</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Let $D$ be the set of all students in the discrete math class. Let $M$, $C$, $F$, $S$, $J$, $R$ be the predicates $M(x) = \text{“x is a math major”}$, $C(x) = \text{“x is a cs major”}$, $F(x) = \text{“x is a freshman”}$, $S(x) = \text{“x is a sophomore”}$, $J(x) = \text{“x is a junior”}$, $R(x) = \text{“x is a senior”}$. Express each of the following statements using quantifiers, and determine if it is true.

1. There is a student in the class who is a junior
   $$\exists x \in D.J(x) \quad \text{True}$$

2. All students in the class are CS majors
   $$\forall x \in D.C(x) \quad \text{False}$$

3. There is a student in the class who is neither a math major nor a junior
   $$\exists x \in D.\neg M(x) \land \neg J(x) \quad \text{True}$$

4. Every student in the class is either a sophomore or a cs major
   $$\forall x \in D.S(x) \lor C(x) \quad \text{False}$$

5. Every senior in the class is a CS major.
   $$\forall x \in D.R(x) \rightarrow C(x) \quad \text{True}$$
11 Integers [8 points]

Let $Q(x, y)$ be the statement “$x + y = x - y$”. Determine which of the following statements are true or false when the variables range of the integer numbers.

1. T ✔ $Q(1, 1)$
2. T ✔ $Q(2, 0)$
3. T ✔ $\exists x. Q(x, 2)$
4. T ✔ $\forall x \exists y. Q(x, y)$
5. T ✔ $\forall y. Q(1, y)$
6. T ✔ $\exists x \exists y. Q(x, y)$
7. T ✔ $\exists x. Q(x, x)$
8. T ✔ $\exists y \forall x. Q(x, y)$

12 Negation [6 points]

Write the negation of each of the following statements without using $\neg$. (You can use $<, \leq, >, \geq, =, \forall, \exists, \land, \lor$, and $\bar{P}(x)$ for the negation of $P(x)$.)

1. $\exists x. 0 \leq x \leq 1 \land P(x)$
   $\forall x. (0 > x) \lor (x > 1) \lor \bar{P}(x)$
2. $\forall x. \exists y. (x < y < x^2) \rightarrow P(x)$
   $\exists x. \forall y. (x < y < x^2) \land \bar{P}(x)$
3. $\forall x. < 10. P(x)$
   $\exists x. < 10. \bar{P}(x)$
13 Propositional inference [8 points]

Food on campus is either good or bad, but not both. Food is bad only if you go to Soupway or CafeLA. If food is good, then you eat a lot. If you go to CafeLA, you gain weight. You don’t eat a lot, and don’t gain any weight. Use the abbreviations below and the rules of inference (modus ponens, modus tollens, generalization, specialization, conjunction, elimination, transitivity, proof by cases, contradiction) to prove that you go to Soupway. State which rule you use at each step.

1. G = food is good
2. B = food is bad
3. S = You go to Soupway
4. C = You go to CafeLA
5. E = You eat a lot
6. W = You gain weight

\[ \begin{align*}
1 & \quad G \lor B \\
2 & \quad \neg(G \lor B) \\
3 & \quad B \rightarrow (S \lor C) \\
4 & \quad G \rightarrow E \\
5 & \quad C \rightarrow W \\
6 & \quad \neg E \\
7 & \quad \neg W
\end{align*} \]

\[ \begin{align*}
8 & \quad \neg C \quad \text{Modus Tollens (5, 7)} \\
9 & \quad \neg G \quad \text{Modus Tollens (4, 6)} \\
10 & \quad B \quad \text{Elimination (1, 9)} \\
11 & \quad S \lor C \quad \text{Modus Ponens (3, 10)} \\
12 & \quad S \quad \text{Elimination (8, 11)}
\end{align*} \]
14 Deduction (predicate logic) [7 points]

Formulate the following statements using predicate logic and the predicates: 
F(x)= “x is a Ferrari”, H(x)= “x goes at 200mph”, G(x)= “x is in the garage”, M(x)= “x is mine”, P(x)= “x makes 50 mpg”.

1. No cars except Ferrari go at 200mph
   \[ \forall x. \neg F(x) \rightarrow \neg H(x) \]

2. All cars in the garage are mine
   \[ \forall x. G(x) \rightarrow M(x) \]

3. Ferrari cars do not make 50 miles per gallon
   \[ \forall x. F(x) \rightarrow \neg P(x) \]

4. None of my cars is slower than 200mph
   \[ \forall x. M(x) \rightarrow H(x) \]

Write one conclusion that you can draw from the above premises. Formulate your conclusion using both predicate logic, and English, and briefly justify your answer.

There are many conclusions. Here is one:
All my cars are Ferrari.
\[ \forall x. M(x) \rightarrow F(x) \]
Using transitive law of 4 and 1.