CSE 105 - Midterm

October 30, 2008

- You have 80 minutes to complete the exam.
- You may use your book and notes.
- You may not use any electronic devices (e.g., computer, cell phone, etc)
- Do not talk to anyone during the exam.
- Write your answers in the space provided. You may use the backs of the pages as scratch paper.
- There are 105 possible points. Thus, it is possible to get 5 extra credit points.

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Email:

<table>
<thead>
<tr>
<th>Problem 1</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem 2</td>
<td></td>
</tr>
<tr>
<td>Problem 3</td>
<td></td>
</tr>
<tr>
<td>Problem 4</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
</tr>
</tbody>
</table>
Problem 1. (25 Points)

Consider $\Sigma = \{0, 1, <\}$ and

$$F = \{ w < x \mid w, x \in \{0, 1\}^* \text{ and } w \text{ is less than } x \text{ when interpreted as binary numbers} \}.$$ 

So, for example, $0001 < 111 \in F$ since $0001 = 1$ is less than $111 = 7$. Show that $F$ is not regular.
Problem 2. (30 Points)
Let $\Sigma = \{0, 1\}$ and let

$L = \{w \mid w \text{ contains two or more occurrences of the substring } 01, \text{ OR } w \text{ is the empty string}\}$.

a.) Show that $L$ is regular by giving an NFA that recognizes it.

b.) Give a regular expression for $L$. 
Problem 3. (20 Points)
Let $A$ be a regular language with alphabet $\Sigma = \{a, b\}$. Show that $A$ can be recognized by an NFA $N$ such that the start state of $N$ has exactly one outgoing transition arrow labeled with character $a$. (The start state of $N$ may have many transition arrows labeled $b$ or $\epsilon$, but must have exactly 1 labeled with $a$.)
Problem 4. (30 Points)
Prove or disprove the following. (You should also write “Prove” or “Disprove” at the top of your answer so we know which you are trying to show.)

a.) “If a language \( L \) is recognized by both DFAs \( M_1 \) and \( M_2 \), then \( M_1 \) and \( M_2 \) must have the same number of states.”

b.) “All non-regular languages contain an infinite number of strings.”

c.) “If \( A \) and \( B \) are both non-regular languages, then the intersection \( A \cap B \) is also non-regular.”