CSE 105 Homework 5

Due March 12

• You may (and are encouraged to) discuss the problems with other students, but what you hand in must be your own work.

• If you do not understand a problem or find a mistake, please email William at wgmatthews@cs.ucsd.edu.

• In addition to these problems, we recommend doing Sipser problems 4.18, 4.19, 4.28, 5.1, 5.9, 5.20, 5.22, 5.23, 5.33 for more practice. However, you should not hand in your solutions to these problems.

• On any problems where you are constructing TMs, make sure it is very clear why the TM solves the problem. This can either be because it is obvious from the description of the TM, or from additional explanation.

• Typed homework solutions will receive 5% extra credit.

1 Decidable Languages

Let A, B, C be recognizable languages over an alphabet Σ, such that A ∪ B ∪ C = Σ* and A ∩ B = ∅, A ∩ C = ∅, and B ∩ C = ∅. Show that A is not only recognizable, but also decidable.

2 Another Decidable Language

Show that the following language is decidable.

L = {⟨M⟩ | M is a DFA and there exists some string w such that both w and wR are in L(M).}

3 Undecidable Languages

Prove that each of the following languages is undecidable. (This problem will be worth twice the points of the others.)

a)

NEVERLOOPTM = {⟨M⟩ | M is a TM and M halts on all inputs.}

b)

VERIFYTM = {⟨M, w, r⟩ | M is a TM, r ∈ {A, R, L}, and

r = A if M accepts w; r = R if M rejects w; and r = L if M loops on w.}

c)

KSTRINGS = {⟨M, k⟩ | M is a TM and M accepts exactly k strings.}

4  Miscellanea

Prove or disprove each of the following statements.

a) Every recognizable language can be recognized by a TM that either accepts or loops, but never rejects.

b) If a language is decidable, then every proper subset of that language is decidable.

c) If $A \leq_m B$ and $B$ is a regular language then $A$ is a regular language.