CSE 105 Homework 1

Due January 15

- You may (and are encouraged to) discuss the problems with other students, but what you hand in must be your own work.
- If you do not understand a problem or find a mistake, please email William at wgmattew@cs.ucsd.edu.
- In addition to these problems, we recommend doing Sipser problems 1.4, 1.5, 1.7, 1.11, 1.16, 1.31, 1.32, 1.40 for more practice. However, you should not hand in your solutions to these problems.
- If you give an NFA or DFA as an answer, remember to give a correctness argument explaining why the NFA or DFA is correct.
- Typed homework solutions will receive 5% extra credit.

1 Set Review

Consider the sets $A = \{1, 2, 3\}$, $B = \{2, 4, 6\}$, and $C = \{1, 3\}$.

a) What is $A \cap C$?

b) What is $B \cup C$?

c) What is the power set of $B$?

d) What is $A \times C$?

e) List all of the ways that we can correctly fill in the blanks in $\quad \subseteq \quad$ and $\quad \subset \quad$ with the sets $A, B, \text{ and } C$.

2 Constructing a DFA

Let $\Sigma = \{0, 1\}$ and consider the language $L$ consisting of strings $x \in \Sigma^*$ which satisfy the following three properties:

- $x$ has no leading 0’s.

- When viewed as a binary number, $x$ is even.

- When viewed as a binary number, $x$ is not divisible by 4.

For example, $10010 \in L$ since it has no leading 0’s and is the binary representation of 18 which is even and not divisible by 4. On the other hand, $00010, 101, \text{ and } 100000$ are not in $L$ because they violate the first, second, and third conditions respectively.

Give a DFA that recognizes $L$. This can be done with a DFA with 5 states, but you should make sure that yours has no more than 10 states.
3 Constructing an NFA

Let $\Sigma = \{a, b\}$ and consider the language $L = a^*b \cup b^*$

a) Give an NFA that recognizes $L$. Your NFA should have at most 4 states.

b) Give a DFA that recognizes $L$. Your DFA should have at most 16 states.

4 Closure

Let $A$ be a language. Define the operation $\text{splittable}$ as follows:

$$A^{\text{splittable}} = \{w_1w_2 \mid w_1 \in A, w_2 \in A, \text{ and } w_1w_2 \in A\}$$

Show that the class of regular languages is closed under the $\text{splittable}$ operation.

5 More Regular Languages

Let $\Sigma = \{0, 1\}$. Consider the following language:

$$C = \{a_1b_1a_2b_2 \ldots a_kb_k \mid a_i, b_i \in \Sigma \text{ and when viewed as binary numbers, } |a_1a_2\ldots a_k - b_1b_2\ldots b_k| = 1\}$$

For example, $10010101 \in C$ since $|1000 - 0111| = 1$, but $11000100 \notin C$ since $|1000 - 1010| = 2$.

Show that $C$ is regular. Hint: This problem is similar to problems 1.32-1.34 in Sipser.