1. Suppose \( f : A \rightarrow B \). Define the inverse set as
\[
f^{-1}(b) = \{ a \in A \mid f(a) = b \} \quad \text{for } b \in B.
\]
Note that \( f^{-1}(b) \) is a set. Prove that the collection of these inverse sets
\[
\{ f^{-1}(b) \mid b \in B \}
\]
is a partition of \( A \). \textit{Hint:} You need to show two properties. First, prove that for all \( a \in A \), there is some \( b \) such that \( a \in f^{-1}(b) \). Second, show that each \( a \in A \) belongs to \textit{only one} set \( f^{-1}(b) \) (and hence the sets \( f^{-1}(b) \) must be disjoint).
2. Suppose $A$ is countable and $B$ is uncountable. Is $A \cap B$ countable? Is $A \cup B$ countable? Why?
3. Prove by induction that

\[ \sum_{i=1}^{n} i(i + 1) = \frac{n(n + 1)(n + 2)}{3} \]  
for \( n \geq 1 \).
4. Define the *fibonacci* sequence as

\[
\begin{align*}
    f_1 &= 1 \\
    f_2 &= 1 \\
    f_i &= f_{i-1} + f_{i-2} & \text{for } i \geq 3.
\end{align*}
\]

Using induction, prove that \(f_{3k}\) is even for all \(k \geq 1\) (e.g. \(f_3\) is even, \(f_6\) is even, etc.).
5. Let $d$ and $k$ be positive integers. Define a relation $R$ on $\mathbb{Z}$ as

$$(x, y) \in R \text{ if } d | (x^k - y^k).$$

Prove that $R$ is an equivalence relation.
6. Prove that for any sets $A, B, C$

$$(A - B) \cap (A - C) = A - (B \cup C).$$
7. How many elements does \( \mathcal{P}(\mathcal{P}(\mathcal{P}(\emptyset))) \) have? (\( \mathcal{P}(A) \) denotes the powerset of \( A \)).
8. A relation $R$ on $A$ is circular if for all $x, y, z \in A$, $xRy$ and $yRz$ implies $zRx$. Show that a reflexive circular relation is an equivalence relation.
9. Suppose that \( f : A \rightarrow B, g : B \rightarrow C \) are both onto. Prove that \( g \circ f \) is onto.