Reflectance & Lighting

Computer Vision I
CSE252A
Lecture 6
Last lecture in a nutshell

- Need for lenses (blur from pinhole)
- Thin lens equation
- Distortion and aberrations
- Vignetting
Radiometry

• Read Chapter 4 of Ponce & Forsyth
• Homework 1 Assigned

Outline

– Solid Angle
– Irradiance
– Radiance
– BRDF
– Lambertian/Phong BRDF
Solid Angle

• By analogy with angle (in radians), the solid angle subtended by a region at a point is the area projected on a unit sphere centered at that point.

• The solid angle subtended by a patch area $dA$ is given by

$$d\omega = \frac{dA \cos \theta}{r^2}$$
Radiance

• Power is energy per unit time (watts)

• Radiance: Power traveling at some point in a specified direction, per unit area perpendicular to the direction of travel, per unit solid angle

• Symbol: $L(x, \theta, \phi)$

• Units: watts per square meter per steradian: $w/(m^2\text{sr})$
Radiance

- Power is energy per unit time

- Radiance: Power traveling at some point in a specified direction, per unit area perpendicular to the direction of travel, per unit solid angle

- Symbol: \( L(x, \theta, \phi) \)

- Units: watts per square meter per steradian: \( \text{w/(m}^2\text{sr}) \)

\[
L = \frac{P}{(dA \cos \theta)d\omega}
\]

Power emitted from patch, but radiance in direction different from surface normal
Radiance properties

- In free space, radiance is constant as it propagates along a ray
  - Derived from conservation of flux
  - Fundamental in Light Transport.

\[
d\Phi_1 = L_1 d\omega_1 dA_1 = L_2 d\omega_2 dA_2 = d\Phi_2
\]

\[
d\omega_1 = dA_2 / r^2 \quad d\omega_2 = dA_1 / r^2
\]

\[
d\omega_1 dA_1 = \frac{dA_1 dA_2}{r^2} = d\omega_2 dA_2
\]

\[
\therefore L_1 = L_2
\]
Radiance transfer

What is the power received by a small area dA₂ at distance r from a small area dA₁ emitting radiance L?

\[ P = L dA_1 \cos \theta_1 d\omega_{1 \rightarrow 2} \]

From definition of solid angle

\[ d\omega = \frac{dA \cos \theta}{r^2} \]

From definition of radiance

\[ L = \frac{P}{(dA \cos \theta) d\omega} \]
Irradiance

- How much light is arriving at a surface?
- Units of irradiance: Watts/m²
- This is a function of incoming angle.
- A surface experiencing radiance \( L(x, \theta, \phi) \) coming in from solid angle \( d\omega \) experiences **irradiance**:

\[
dE(x) = L(x, \theta, \phi) \cos \theta d\omega
\]

- Crucial property:
  Total **Irradiance** arriving at the surface is given by adding irradiance over all incoming angles 
  Total irradiance is

\[
E(x) = \int L(x, \theta, \phi) \cos \theta d\omega
\]

for **hemisphere**

\[
= \int_{0}^{\pi/2} \int_{0}^{2\pi} L(x, \theta, \phi) \cos \theta \sin \theta d\theta d\phi
\]
Intermezzo:

Camera’s sensor

- Measured pixel intensity is a function of irradiance integrated over
  - pixel’s area
  - over a range of wavelengths
  - For some time

\[
I = \int_{t} \int_{\lambda} \int_{x} \int_{y} E(x, y, \lambda, t) s(x, y) q(\lambda) dy dx dl \lambda dt
\]

- Ideally, it’s a linear function of the radiance, but the camera response \( C(.) \) may not be linear

\[
I = C \left( \int_{t} \int_{\lambda} \int_{x} \int_{y} E(x, y, \lambda, t) s(x, y) q(\lambda) dy dx dl \lambda dt \right)
\]
Image sensor

Two types:

1. CCD
2. CMOS
CCD

separate photo sensor at regular positions
no scanning
charge-coupled devices (CCDs):
: interline transfer and frame transfer

- photosensitive
- storage
CMOS

Each photo sensor has its own amplifier
  More noise (reduced by subtracting ‘black’ image)
  Lower sensitivity (lower fill rate)
Uses standard CMOS technology
  Allows to put other components on chip
  ‘Smart’ pixels
CCD vs. CMOS

CCD:
- Mature technology
- Specific technology
- High production cost
- High power consumption
- Higher fill rate
- Blooming
- Sequential readout

CMOS:
- Recent technology
- Standard IC technology
- Cheap
- Low power
- Less sensitive
- Per pixel amplification
- Random pixel access
- Smart pixels
- On chip integration with other components
Color Cameras

We consider 3 concepts:

1. Prism (with 3 sensors)
2. Filter mosaic
3. Filter wheel

... and X3
Prism color camera

Separate light in 3 beams using dichroic prism
Requires 3 sensors & precise alignment
Good color separation
Filter mosaic

Coat filter directly on sensor

Demosaicing (obtain full colour & full resolution image)
Filter wheel

Rotate multiple filters in front of lens
Allows more than 3 colour bands

Only suitable for static scenes
# Prism vs. mosaic vs. wheel

<table>
<thead>
<tr>
<th>approach</th>
<th>Prism</th>
<th>Mosaic</th>
<th>Wheel</th>
</tr>
</thead>
<tbody>
<tr>
<td># sensors</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Separation</td>
<td>High</td>
<td>Average</td>
<td>Good</td>
</tr>
<tr>
<td>Cost</td>
<td>High</td>
<td>Low</td>
<td>Average</td>
</tr>
<tr>
<td>Framerate</td>
<td>High</td>
<td>High</td>
<td>Low</td>
</tr>
<tr>
<td>Artefacts</td>
<td>Low</td>
<td>Aliasing</td>
<td></td>
</tr>
<tr>
<td>Bands</td>
<td>3</td>
<td>3</td>
<td>3 or more</td>
</tr>
<tr>
<td></td>
<td>High-end cameras</td>
<td>Low-end cameras</td>
<td>Scientific applications</td>
</tr>
</tbody>
</table>
“newer” color CMOS sensor
Foveon’s X3

better image quality

smarter pixels

Pixel Size: 1x1
Pixel Size: 4x4
Pixel Size: 1x2

VPS Enables a Foveon X3 image sensor to be addressed in variable resolutions.
What is image irradiance $E$ for a radiance $L$ emitted from a point $P$?
Let $\delta \omega$ be the solid angle subtended by $\delta A$ (or $\delta A'$) from the center of the lens.

$$\delta \omega = \frac{\delta A' \cos \alpha}{(z'/\cos \alpha)^2} = \frac{\delta A \cos \beta}{(z/\cos \alpha)^2}$$

$$\frac{\delta A}{\delta A'} = \frac{\cos \alpha}{\cos \beta} \left(\frac{z}{z'}\right)^2$$
Let \( \Omega \) be the solid angle subtended by the lens from \( P \).

\[
\delta \omega = \frac{\delta A' \cos \alpha}{(z' / \cos \alpha)^2} = \frac{\delta A \cos \beta}{(z / \cos \alpha)^2}
\]

\[
\frac{\delta A}{\delta A'} = \frac{\cos \alpha}{\cos \beta} \left( \frac{z}{z'} \right)^2
\]

\[
\Omega = \frac{\pi}{4} \frac{d^2 \cos \alpha}{(z / \cos \alpha)^2} = \frac{\pi}{4} \left( \frac{d}{z} \right)^2 \cos \alpha^3
\]
The power \( \delta P \) emitted from the patch \( \delta A \) with radiance \( L \) and falling on the lens is:

\[
\delta P = L \Omega \delta A \cos \beta = \frac{\pi}{4} \left( \frac{d}{z} \right)^2 L \delta A \cos^3 \alpha \cos \beta
\]
Radiometry of thin lenses

\[ \delta\omega = \frac{\delta A' \cos \alpha}{(z' / \cos \alpha)^2} = \frac{\delta A \cos \beta}{(z / \cos \alpha)^2} \]

\[ \Omega = \frac{\pi}{4} \frac{d}{z} \cos \alpha \]

\[ \delta P = L\Omega \delta A \cos \beta = \frac{\pi}{4} \frac{d}{z} L\delta A \cos^3 \alpha \cos \beta \]

\[ E = \frac{\delta P}{\delta A'} = \frac{\pi}{4} \left( \frac{d}{z} \right)^2 L \frac{\delta A}{\delta A'} \cos \alpha \cos \beta \]

\[ E = \left[ \frac{\pi}{4} \left( \frac{d}{z} \right)^2 \cos^4 \alpha \right] L \]

- \( E \): Image irradiance
- \( L \): emitted radianc
- \( d \): Lens diameter
- \( z \): depth
- \( \alpha \): Angle of patch from optical axis