Corners + Stereo

Computer Vision I
CSE252A
Lecture 14
Announcements

• HW3 to be assigned soon. Stereo
Edges
2D Edge Detection: Canny

1. Filter out noise
   - Use a 2D Gaussian Filter. \( J = I \otimes G \)

2. Take a derivative
   - Magnitude and direction of the gradient:

\[
\nabla J = (J_x, J_y) = \left( \frac{\partial J}{\partial x}, \frac{\partial J}{\partial y} \right) \text{ is the Gradient}
\]

\[
\| \nabla J \| = \sqrt{J_x^2 + J_y^2}
\]
Non-maximum suppression

At q, we have a maximum if the value is larger than those at both p and at r. Interpolate to get these values.
Assume the marked point is an edge point. Then we construct the tangent to the edge curve (which is normal to the gradient at that point) and use this to predict the next points (here either r or s).
Hysteresis

- Track edge points by starting at point where gradient magnitude $> \tau_{\text{high}}$.
- Follow edge in direction orthogonal to gradient.
- Stop when gradient magnitude $< \tau_{\text{low}}$.
  - i.e., use a high threshold to start edge curves and a low threshold to continue them.
Formula for Finding Corners

Let \[ I_x = \frac{\partial I}{\partial x}, \text{and } I_y = \frac{\partial I}{\partial y} \]

Sum over a small region, the hypothetical corner

\[ C = \begin{bmatrix}
\sum I_x^2 \\
\sum I_x I_y \\
\sum I_x I_y \\
\sum I_y^2 
\end{bmatrix} \]

Gradient with respect to x, times gradient with respect to y

Matrix is symmetric

WHY THIS?
First, consider case where:

\[ C = \begin{bmatrix}
\sum I_x^2 & \sum I_x I_y \\
\sum I_x I_y & \sum I_y^2
\end{bmatrix} = \begin{bmatrix}
\lambda_1 & 0 \\
0 & \lambda_2
\end{bmatrix} \]

This means all gradients in neighborhood are:

(k,0) or (0, c) or (0, 0) (or off-diagonals cancel).

What is region like if:

1. \( \lambda_1 = 0 \)?
2. \( \lambda_2 = 0 \)?
3. \( \lambda_1 = 0 \) and \( \lambda_2 = 0 \)?
4. \( \lambda_1 > 0 \) and \( \lambda_2 > 0 \)?
General Case:

From Linear Algebra, it follows that

\[ C = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R \]

since C is symmetric. So every case is like the one on the last slide.
So, to detect corners

- Filter image.
- Compute the gradient everywhere.
- We construct C in a window of some size.
- Use linear algebra to find $\lambda_1$ and $\lambda_2$.
- If $\lambda_1$ and $\lambda_2$ are both big, we have a corner.
  1. Let $e(u,v) = \min(\lambda_1(u,v), \lambda_2(u,v))$
  2. $(u,v)$ is a corner if it is a local maximum of $e(u,v)$ and $e(u,v) > \tau$
Corner Detection Sample Results

Threshold = 25,000

Threshold = 10,000

Threshold = 5,000
Stereo
Binocular Stereopsis: Mars

Given two images of a scene where relative locations of cameras are known, estimate depth of all common scene points.

Two images of Mars
An Application: Mobile Robot Navigation


The INRIA Mobile Robot, 1990.


Courtesy O. Faugeras and H. Moravec.
Commercial Stereo Heads

Trinocular stereo

Binocular stereo
Stereo can work well
Need for correspondence
Triangulation
Stereo Vision Outline

• Offline: Calibrate cameras & determine “epipolar geometry”

• Online
  1. Acquire stereo images
  2. Rectify images to convenient epipolar geometry
  3. Establish correspondence
  4. Estimate depth
BINOCULAR STEREO SYSTEM

Estimating Depth

DISPARITY

\( (X_L - X_R) \)

\[
Z = \left( \frac{f}{X_L} \right) X \\
Z = \left( \frac{f}{X_R} \right) (X - d)
\]

\[
\left( \frac{f}{X_L} \right) X = \left( \frac{f}{X_R} \right) (X - d) \\
X = \frac{(X_L d)}{(X_L - X_R)}
\]

\[
X = \frac{d X_L}{(X_L - X_R)}
\]

\[
Z = \frac{d f}{(X_L - X_R)}
\]

\[
X_L = f(X/Z) \\
X_R = f((X - d)/Z)
\]

(Adapted from Hager)
Reconstruction: General 3-D case

Given two image measurements $p$ and $p'$, estimate $P$.

• Linear Method:
  find $P$ such that
  \[
  \begin{align*}
  p \times \mathcal{M}P &= 0 \\
  p' \times \mathcal{M}'P &= 0
  \end{align*}
  \iff
  \left( \begin{bmatrix} p_x \\ p'_x \end{bmatrix} \mathcal{M} \right) P = 0
  \]

• Non-Linear Method: find $Q$ minimizing
  \[d^2(p, q) + d^2(p', q')\]
  where $q = MQ$ and $q' = M'Q$
Two Approaches

1. Feature-Based
   - From each image, process “monocular” image to obtain cues (e.g., corners, lines).
   - Establish correspondence between

2. Area-Based
   - Directly compare image regions between the two images.
Human Stereopsis: Binocular Fusion

How are the correspondences established?

Julesz (1971): Is the mechanism for binocular fusion a monocular process or a binocular one??
• There is anecdotal evidence for the latter (camouflage).

• Random dot stereograms provide an objective answer
Random Dot Stereograms
Random Dot Stereograms
A Cooperative Model (Marr and Poggio, 1976)
Random Dot Stereograms
Where does a point in the left image match in the right image?
Need for correspondence
• Potential matches for \( p \) have to lie on the corresponding epipolar line \( l' \).

• Potential matches for \( p' \) have to lie on the corresponding epipolar line \( l \).
• Epipolar Plane
• Epipoles
• Epipolar Lines

• Baseline
Family of epipolar Planes

Family of planes $\pi$ and lines $l$ and $l'$
Intersection in $e$ and $e'$
Epipolar Constraint: Calibrated Case

\[ \overrightarrow{OP} \cdot [\overrightarrow{OO'} \times \overrightarrow{O'P'}] = 0 \quad \Rightarrow \quad \mathbf{p} \cdot [\mathbf{t} \times (\mathbf{Rp}')] = 0 \quad \text{with} \quad \begin{cases} \mathbf{p} = (u, v, 1)^T \\ \mathbf{p}' = (u', v', 1)^T \\ \mathbf{M} = (\mathbf{I} \quad \mathbf{0}) \\ \mathbf{M}' = (\mathbf{R}^T, -\mathbf{R}^T \mathbf{t}) \end{cases} \]

Essential Matrix
(Longuet-Higgins, 1981)

\[ \mathbf{p}^T \mathbf{E} \mathbf{p}' = 0 \quad \text{with} \quad \mathbf{E} = [\mathbf{t} \times] \mathbf{R} \]
Properties of the Essential Matrix

\[ \mathbf{p}^T \mathbf{E} \mathbf{p}' = 0 \quad \text{with} \quad \mathbf{E} = [\mathbf{t}_x] \mathbf{R} \]

- \( \mathbf{E} \mathbf{p}' \) is the epipolar line associated with \( \mathbf{p}' \).
- \( \mathbf{E}^T \mathbf{p} \) is the epipolar line associated with \( \mathbf{p} \).
- \( \mathbf{E} \mathbf{e}' = 0 \) and \( \mathbf{E}^T \mathbf{e} = 0 \).
- \( \mathbf{E} \) is singular.
- \( \mathbf{E} \) has two equal non-zero singular values (Huang and Faugeras, 1989).