Assignment #1
(Due date: 2/1/07)

1. F&P 1.2 [2pts]
2. F&P 1.3 [3pts]
3. Thin Lens Equation [3pts]

An illuminated arrow forms a real inverted image of itself at a distance \( w = 40 \) cm, measured along the optic axis of a convex thin lens (see figure 1). The image is just half the size of the object.

a. How far from the object must the lens be placed?

b. What is the focal length of the lens?

c. If the index of refraction for the lens is \( \eta = 1.5 \), what is the lens radius?

d. Suppose the arrow moves 5 cm to the right while the lens and image plane remain the fixed. This will result in an out of focus image; what is the radius of the corresponding blur circle formed from the tip of the arrow on the image plane assuming the diameter of the lens is \( d \)?

![Figure 1: Problem 3. setup.](image)

4. F&P 2.5 [2pts]
5. F&P 2.8 [3pts]
6. Affine Pose Estimation [3pts]

An affine camera transforms 3d (homogeneous) points $\mathbf{x} = (x, y, z, 1)^T$ according to,

$$\mathbf{x}' = M\mathbf{x}$$

where

$$M = \begin{pmatrix} a & b & c & d \\ e & f & g & h \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

is the camera matrix and $\mathbf{x}' = (x', y', 1)^T$ is the mapping onto the image plane.

Given a set of corresponding points $\mathbf{x}$ and $\mathbf{x}'$, your task is to recover the affine camera parameters.

a. What is the minimum number of corresponding points needed to recover the camera parameters?

b. Are there any conditions on the positions of these correspondence points for successful recovery of $M$? List two degenerate configurations. Can you think of a necessary condition?

c. Running house.m will display an image of an affine transformed house. A set of 8 corresponding points are contained in the matlab variables Xcorr (original points) and Xcorr_ (affine transformed points). Write a matlab function that takes as input Xcorr and Xcorr_ and returns the affine transformation matrix, $M$. What is the recovered affine transformation matrix?

d. Repeat part (c) using only correspondence points 1,3,4, and 6. What happens when you try to estimate $M$? Why?

e. Write a second program that prompts the user to manually click the correspondence points from the plot of the house model shown after running house.m. Using the manually selected correspondences, compute the affine transformation matrix $M$. Do you get the same result as in part (c)? How sensitive is the estimation to errors in the correspondences (experiment by purposely mis-clicking some of the points)?

What to turn in : Answer the questions and print out your code from parts (c) and (e).

7. Irradiance 1 [3pts]

a. Consider a cylinder with radius $r$ and height $h$ whose base is centered at $z = 0$ along the $xy$-plane. If the walls of the cylinder have constant radiance $L$ and the top of the cylinder has constant radiance $2L$, what is the irradiance $E$ at the point $(0, 0, 0)$ assuming that the surface at $(0, 0, 0)$ has a normal vector of $(0, 0, 1)$? (See figure 2).

b. What is the irradiance if the top has constant radiance $L$ (i.e., the entire cylinder has constant radiance $L$)?

![Figure 2: Problem 7. setup.](image-url)

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1The matlab command `ginput` will be useful here.
8. Irradiance 2 [6pts]

Consider a rectangular surface with vertices $(-2, -1, 1)$, $(-2, 1, 1)$, $(2, 1, 1)$, and $(2, -1, 1)$. If the radiance on the surface is $L(x^2 + y^2 + 1)$, what is the irradiance arriving at position $(0, 0, 0)$ with normal vector $(0, 0, 1)$? (See figure 3).

Figure 3: Problem 8. setup.