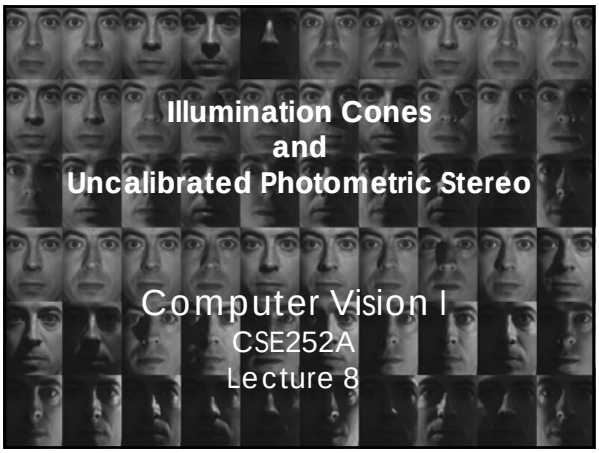


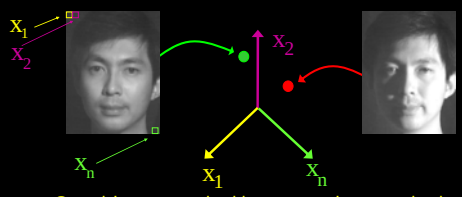
Illumination Cones and Uncalibrated Photometric Stereo

Computer Vision I
CSE252A
Lecture 8

What is the set of images of an object under all possible lighting conditions?

In answering this question, we'll arrive at a method for reconstructing surface shape w/ unknown lighting.

The Space of Images

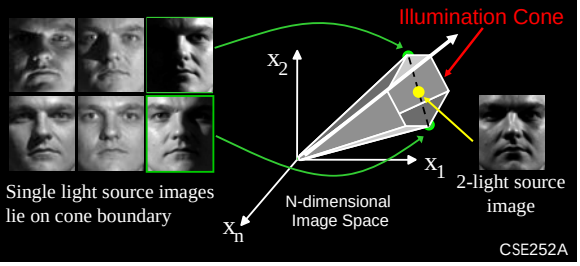


- Consider an n-pixel image to be a point in an n-dimensional space, $\mathbf{x} \in R^n$.
- Each pixel value is a coordinate of **x**.
- Many results will apply to linear transformations of image space (e.g. filtered images)
- Other image representations (e.g. Cayley-Klein spaces, See Koenderink's "pixel f#@king paper")

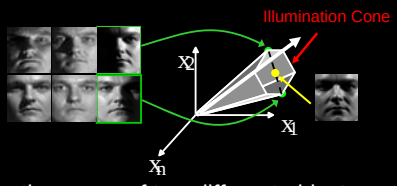
The Illumination Cone

Theorem: *The set of images of any object in fixed posed, but under all lighting conditions, is a* **convex cone** *in the image space.*

(Belhumeur and Kriegman, IJCV, 98)



Some natural ideas & questions

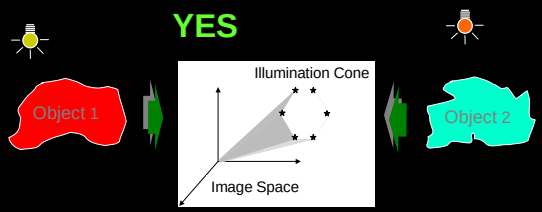


- Can the cones of two different objects intersect?
- Can two different objects have the same cone?
- How big is the cone?
- How can cone be used for recognition?

Do Ambiguities Exist?

Can two objects of differing shapes produce the same illumination cone?

YES



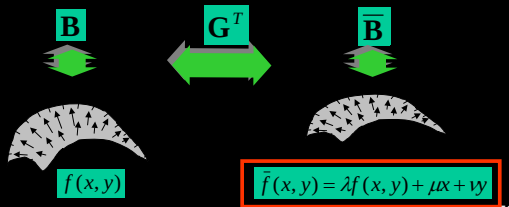
Do Ambiguities Exist? Yes

- Cone is determined by linear subspace **L**
- The columns of **B** span **L**
- For any **A** ∈ GL(3), **B* = BA** also spans **L.**
- For any image of **B** produced with light source **S**, the same image can be produced by lighting **B*** with $\mathbf{S^*=A^{-1}S}$ because
  $\mathbf{X = B^*S^* = B\,AA^{-1}S = BS}$
- When we estimate B using SVD, the rows are NOT generally normal * albedo.
-

GBR Transformation

Only Generalized Bas-Relief transformations satisfy the integrability constraint:

$$\mathbf{A} = \mathbf{G}^T = \begin{bmatrix} \lambda & 0 & -\mu \\ 0 & \lambda & -\nu \\ 0 & 0 & 1 \end{bmatrix}^T$$



Generalized Bas-Relief Transformations

Objects differing by a GBR have the same illumination cone.

Without knowledge of light source location, one can only recover surfaces up to GBR transformations.

Uncalibrated photometric stereo

1. Take n images as input, perform SVD to compute B*.
2. Find some A such that B*A is close to integrable.
3. Integrate resulting gradient field to obtain height function f*(x,y).

Comments:

- f*(x,y) differs from f(x,y) by a GBR.
- Can use specularities to resolve GBR for non-Lambertian surface.

What about cast shadows for nonconvex objects?

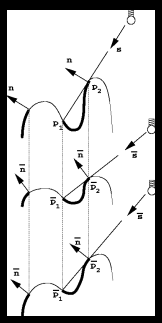
P.P. Reubens in Opticorum Libri Sex, 1613

GBR Preserves Shadows

Given a surface f and a GBR transformed surface f' then for every light source **s** which illuminates f there exists a light source **s'** which illuminates f' such that the **attached** and **cast shadows** are identical.

GBR is the only transform that preserves shadows.

[Kriegman, Belhumeur 2001]



Where'd the moguls go?

- When a convex Lambertian surface is illumination by perfectly diffuse lighting, the resulting image is directly proportional to the albedo.
- For a convex object, the n-dimensional vector of albedos (and image) is contained within the object's cone.
- For two objects with the same albedo pattern but different shape, their cones intersect in the interior.
- Two objects differing by a generalized bas relief transformation have the same cone.

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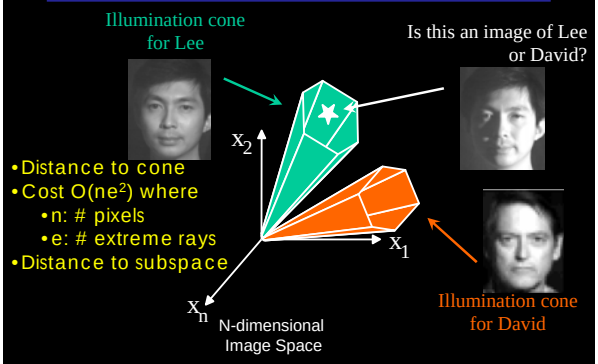
Illumination Cones: Recognition Method



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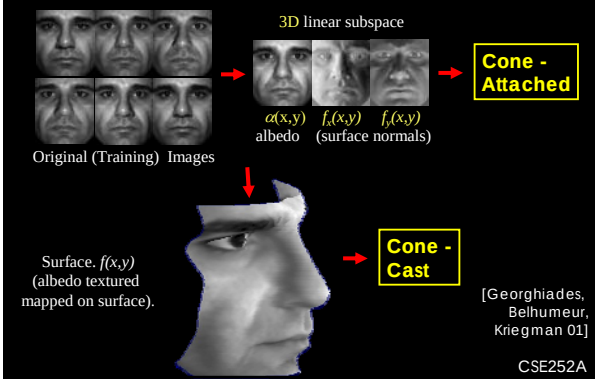
Generating the Illumination Cone



[Georghiades, Belhumeur, Kriegman 01]

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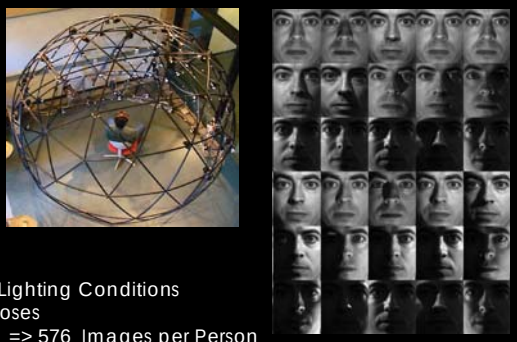
Yale Face Database B



64 Lighting Conditions
9 Poses
=> 576 Images per Person

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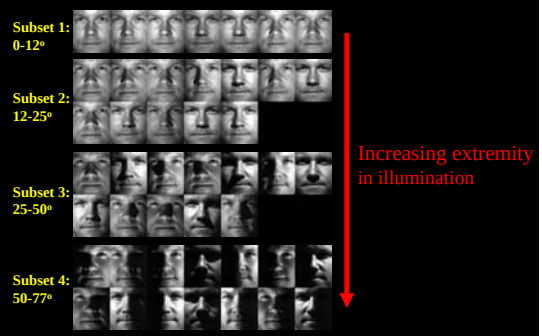
Face Recognition: Test Subsets



Test images divided into 4 subsets depending on illumination.

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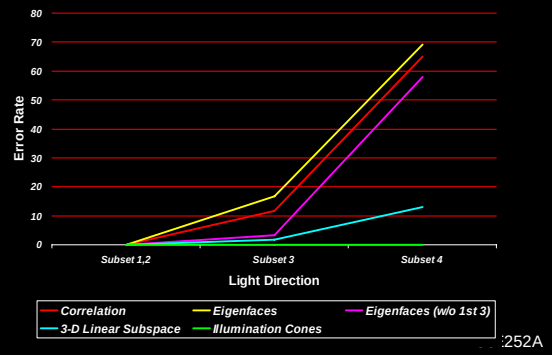
Geodesic Dome Database - Frontal Pose

[Georghiades, Belhumeur, Kriegman 01]



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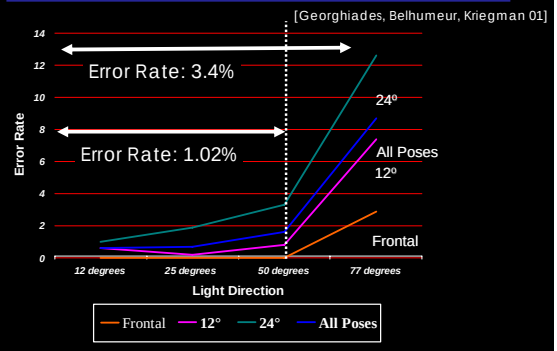
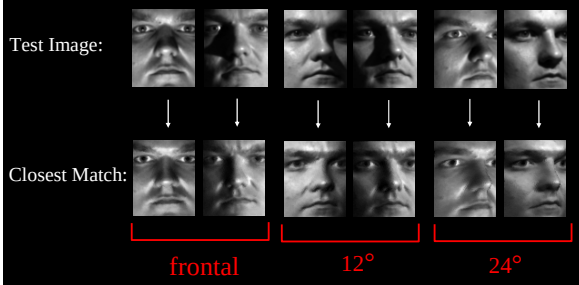
Illumination Cone Face Recognition Result: Pose and Lighting



Closest Sample to Test Image: Examples



Illumination & Image Set

- Lack of illumination invariants
  [Chen, Jacobs, Belhumeur 98]
- Set of images of Lambertian surface w/o shadowing is 3-D linear subspace
  [Moses 93], [Nayar, Murase 96], [Shashua 97]
- Empirical evidence that set of images of object is well-approximated by a low-dimensional linear subspace
  [Hallinan 94], [Epstein, Hallinan, Yuille 95]
- Illumination cones
  [Belhumeur, Kriegman 98]
- Spherical harmonics lighting & images
  [Basri, Jacobs 01], [Ramamoorthi, Hanrahan 01]
- Analytic PCA of image over lighting
  [Ramamoorthi 02]

Some subsequent work

1. "Face Recognition Under Variable Lighting using Harmonic Image Exemplars," Zhang, Samaras, CVPR03
2. "Clustering Appearances of Objects Under Varying Illumination Conditions," Ho, Lee, Lim, Kriegman, CVPR 03
3. "Low-Dimensional Representations of Shaded Surfaces under Varying Illumination, Nillius, Eklundh, CVPR03
4. "Using Specularities for Recognition," Osadchy, Jacobs, Ramamoorthi, ICCV 03