Photometric Stereo
Recap + Lambertian

Computer Vision I
CSE252A
Lecture 8a

Announcements
• HW1 due today

Coordinate system

Reflectance map

For known BRDF, fix light source direction/strength and projection direction
1. Then image irradiance is a function of only the direction of the surface normal.
2. In gradient space, we have \( E(p,q) \) which is known as the reflectance map

Reflectance Map of Lambertian Surface

Two Light Sources
Two reflectance maps

Third image would disambiguate match
Recovering the surface $f(x,y)$

Many methods: Simplest approach
1. From estimate $n = (n_x, n_y, n_z)$, $p = n_x/n_z$, $q = n_y/n_z$
2. Integrate $p = df/dx$ along a row $(x,0)$ to get $f(x,0)$
3. Then integrate $q = df/dy$ along each column starting with value of first row

What might go wrong?

- Height $z(x,y)$ is obtained by integration along a curve from $(x_0, y_0)$.
- If one integrates the derivative field along any closed curve, one expects to get back to the starting value.
- Might not happen because of noisy estimates of $(p, q)$

Integrability

If $f(x,y)$ is the height function, we expect that

\[
\frac{\partial f}{\partial y} = \frac{\partial f}{\partial x} \frac{\partial y}{\partial x} 
\]

In terms of estimated gradient space $(p, q)$, this means:

\[
\frac{\partial p}{\partial y} = \frac{\partial q}{\partial x}
\]

But since $p$ and $q$ were estimated independently at each point as intersection of curves on three reflectance maps, equality is not going to exactly hold

II. Photometric Stereo:
Lambertian Surface, Known Lighting

Lambertian Photometric stereo

- If the light sources $s_1$, $s_2$, and $s_3$ are known, then we can recover $b$ from as few as three images. (Photometric Stereo: Silver 80, Woodham81).

\[
\begin{bmatrix} 
  e_1 & e_2 & e_3 \\
  s_1 & s_2 & s_3
\end{bmatrix} = b^T \begin{bmatrix} 
  s_1 & s_2 & s_3
\end{bmatrix}
\]

- i.e., we measure $e_1$, $e_2$, and $e_3$ and we know $s_1$, $s_2$, and $s_3$. We can then solve for $b$ by solving a linear system.

$\begin{bmatrix} 
  e_1 & e_2 & e_3 \\
  s_1 & s_2 & s_3
\end{bmatrix} = b^T \begin{bmatrix} 
  s_1 & s_2 & s_3
\end{bmatrix}$

- Normal is: $n = b/|b|$, albedo is: $|b|$
What if we have more than 3 Images? Linear Least Squares

\[ [c_1\ c_2\ c_3] = b^T[s_1\ s_2\ s_3] \]

Let the residual be

\[ r = e - Sb \]

Rewrite as

\[ e = Sb \]

where

- \( e \) is \( n \) by 1
- \( b \) is 3 by 1
- \( S \) is \( n \) by 3

Let the residual be

\[ r = e - Sb \]

Squaring this:

\[ r^T r = (c - Sb)^T (c - Sb) \]

\[ = c^T c - 2b^T S^T e + b^T S^T S b \]

\[ \frac{\partial r^2}{\partial b} = 0 \quad \text{zero derivative is a necessary condition for a minimum, or} \]

\[ -2S^T e + 2S^T S b = 0; \]

Solving for \( b \) gives

\[ b = (S^T S)^{-1} S^T e \]

Recovered albedo

Recovered normal field

Surface recovered by integration