

Reflectance & Lighting

Computer Vision I
CSE252A
Lecture 6

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Last lecture in a nutshell

- Need for lenses (blur from pinhole)
- Thin lens equation
- Distortion and aberrations
- Vignetting

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Computer Vision I

Radiometry

- Read Chapter 4 of Ponce & Forsyth
- Homework 1 Assigned

Outline

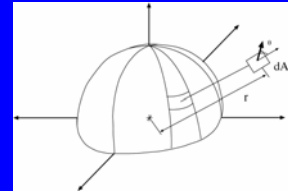
- Solid Angle
- Irradiance
- Radiance
- BRDF
- Lambertian/Phong BRDF

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Solid Angle

- By analogy with angle (in radians), the solid angle subtended by a region at a point is the area projected on a unit sphere centered at that point
- The solid angle subtended by a patch area dA is given by

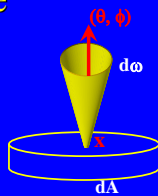
$$d\omega = \frac{dA \cos \theta}{r^2}$$



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Radiance

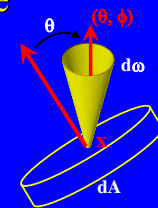
- Power is energy per unit time
- **Radiance:** Power traveling at some point in a specified direction, per unit area perpendicular to the direction of travel, per unit solid angle
- Symbol: $L(x, \theta, \phi)$
- Units: watts per square meter per steradian : $w/(m^2sr)$



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Radiance

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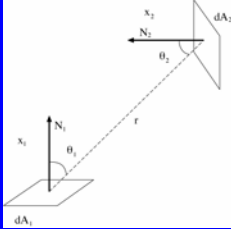
$$L = \frac{P}{(dA \cos \theta) d\omega}$$

Power emitted from patch, but radiance in direction different from surface normal

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Radiance transfer

What is the power received by a small area dA_2 at distance r from a small area dA_1 emitting radiance L ?



From definition of radiance

$$L = \frac{P}{(dA \cos \theta) d\omega}$$

From definition of solid angle

$$d\omega = \frac{dA \cos \theta}{r^2}$$

$$P = L dA_1 \cos \theta_1 d\omega_{1 \rightarrow 2} = \frac{L}{r^2} dA_1 dA_2 \cos \theta_1 \cos \theta_2$$

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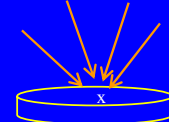
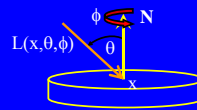
Irradiance

- How much light is arriving at a surface?
- Units of irradiance: Watts/m²
- This is a function of incoming angle.
- A surface experiencing radiance $L(x, \theta, \phi)$ coming in from solid angle $d\omega$ experiences **irradiance**:

$$dE(x) = L(x, \theta, \phi) \cos \theta d\omega$$

- Crucial property: Total **irradiance** arriving at the surface is given by adding irradiance over all incoming angles
- Total irradiance is

$$E(x) = \int_{\text{hemisphere}} L(x, \theta, \phi) \cos \theta d\omega = \int_0^{2\pi} \int_0^{\pi/2} L(x, \theta, \phi) \cos \theta \sin \theta d\theta d\phi$$



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Intermezzo: Camera's sensor

- Measured pixel intensity is a function of irradiance integrated over
 - pixel's area
 - over a range of wavelengths
 - For some time

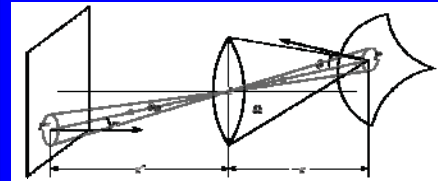
$$I = \int_t \int_\lambda \int_x \int_y E(x, y, \lambda, t) s(x, y) q(\lambda) dy dx d\lambda dt$$

- Ideally, it's linear to the radiance, but the camera response $C(\cdot)$ may not be linear

$$I = C \left(\int_t \int_\lambda \int_x \int_y E(x, y, \lambda, t) s(x, y) q(\lambda) dy dx d\lambda dt \right)$$

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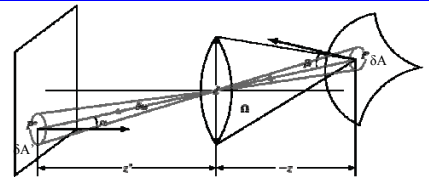
Radiometry of thin lenses



What is image irradiance E for a radiance L emitted from a point P ?

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Radiometry of thin lenses



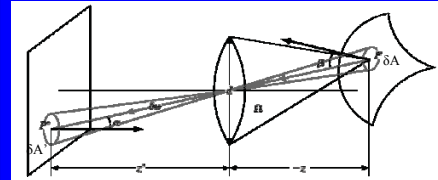
Let $\delta\omega$ be the solid angle subtended by δA (or $\delta A'$) from the center of the lens

$$\delta\omega = \frac{\delta A' \cos \alpha}{(z'/\cos \alpha)^2} = \frac{\delta A \cos \beta}{(z/\cos \alpha)^2}$$

$$\frac{\delta A}{\delta A'} = \frac{\cos \alpha}{\cos \beta} \left(\frac{z'}{z} \right)^2$$

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Radiometry of thin lenses



$$\delta\omega = \frac{\delta A' \cos \alpha}{(z'/\cos \alpha)^2} = \frac{\delta A \cos \beta}{(z/\cos \alpha)^2} \quad \frac{\delta A}{\delta A'} = \frac{\cos \alpha}{\cos \beta} \left(\frac{z'}{z} \right)^2$$

Let Ω be the solid angle subtended by the lens from P .

$$\Omega = \frac{\pi}{4} \frac{d^2 \cos \alpha}{(z/\cos \alpha)^2} = \frac{\pi}{4} \left(\frac{d}{z} \right)^2 \cos \alpha^3$$

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Radiometry of thin lenses

$$\delta\omega = \frac{\delta A' \cos \alpha}{(z'/\cos \alpha)^2} = \frac{\delta A \cos \beta}{(z/\cos \alpha)^2} \quad \frac{\delta A}{\delta A'} = \frac{\cos \alpha}{\cos \beta} \left(\frac{z}{z'}\right)^2$$

$$\Omega = \frac{\pi}{4} \frac{d^2 \cos \alpha}{(z/\cos \alpha)^2} = \frac{\pi}{4} \left(\frac{d}{z}\right)^2 \cos \alpha$$

The power δP emitted from the patch δA with radiance L and falling on the lens is:

$$\delta P = L \Omega \delta A \cos \beta = \frac{\pi}{4} \left(\frac{d}{z}\right)^2 L \delta A \cos^3 \alpha \cos \beta$$

Radiometry of thin lenses

$$\delta\omega = \frac{\delta A' \cos \alpha}{(z'/\cos \alpha)^2} = \frac{\delta A \cos \beta}{(z/\cos \alpha)^2} \quad \frac{\delta A}{\delta A'} = \frac{\cos \alpha}{\cos \beta} \left(\frac{z}{z'}\right)^2$$

$$\Omega = \frac{\pi}{4} \frac{d^2 \cos \alpha}{(z/\cos \alpha)^2} = \frac{\pi}{4} \left(\frac{d}{z}\right)^2 \cos \alpha$$

$$\delta P = L \Omega \delta A \cos \beta = \frac{\pi}{4} \left(\frac{d}{z}\right)^2 L \delta A \cos^3 \alpha \cos \beta$$

$$E = \frac{\delta P}{\delta A'} = \frac{\pi}{4} \left(\frac{d}{z}\right)^2 L \frac{\delta A}{\delta A'} \cos^3 \alpha \cos \beta$$

$$E = \left[\frac{\pi}{4} \left(\frac{d}{z}\right)^2 \cos^4 \alpha \right] L$$

E: Image irradiance
L: emitted radiance
d: Lens diameter
z: depth
 α : Angle of patch from optical axis

Light at surfaces

Many effects when light strikes a surface -- could be:

- transmitted
 - Skin, glass
- reflected
 - mirror
- scattered
 - milk
- travel along the surface and leave at some other point
- absorbed
 - sweaty skin

Assume that

- surfaces don't fluoresce
 - e.g. scorpions, detergents
- surfaces don't emit light (i.e. are cool)
- all the light leaving a point is due to that arriving at that point

BRDF

With assumptions in previous slide

- Bi-directional Reflectance Distribution Function

$$\rho(\theta_{in}, \phi_{in}; \theta_{out}, \phi_{out})$$
- Ratio of incident irradiance to emitted radiance
- Function of
 - Incoming light direction: θ_{in}, ϕ_{in}
 - Outgoing light direction: θ_{out}, ϕ_{out}

$$\rho(x; \theta_{in}, \phi_{in}; \theta_{out}, \phi_{out}) = \frac{L_o(x; \theta_{out}, \phi_{out})}{L_i(x; \theta_{in}, \phi_{in}) \cos \theta_{in} d\omega}$$

The Reflection Equation

$$L_r(x, \omega_r) = \int_{H^2} f_r(x, \omega_i \rightarrow \omega_r) L_i(x, \omega_i) \cos \theta_i d\omega_i$$

CS348B Lecture 10 Pat Hanrahan, Spring 2002

Properties of BRDFs

- Non-negative: $\rho(\theta_{in}, \phi_{in}; \theta_{out}, \phi_{out}) \geq 0$
- Helmholtz Reciprocity Principle:

$$\rho(\theta_{in}, \phi_{in}; \theta_{out}, \phi_{out}) = \rho(\theta_{out}, \phi_{out}; \theta_{in}, \phi_{in})$$
- Total energy leaving a surface must be less than total energy arriving at the surface

$$\int_{\Omega_r} L_r(x, \theta_r, \phi_r) \cos \theta_r d\omega_r \geq \int_{\Omega_i} \left[\int_{\Omega_r} \rho(\theta_i, \phi_i; \theta_r, \phi_r) L_i(x, \theta_i, \phi_i) \cos \theta_i d\omega_i \right] \cos \theta_r d\omega_r$$

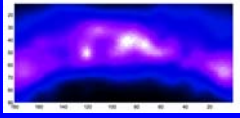
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Surface Reflectance Models

Common Models

- Lambertian
- Phong
- Physics-based
 - Specular [Blinn 1977], [Cook-Torrance 1982], [Ward 1992]
 - Diffuse [Hanrahan, Kreuger 1993]
 - Generalized Lambertian [Oren, Nayar 1995]
 - Thoroughly Pitted Surfaces [Koenderink et al 1999]
- Phenomenological [Koenderink, Van Doorn 1996]

Arbitrary Reflectance





- Non-parametric model
- Anisotropic
- Non-uniform over surface
- BRDF Measurement [Dana et al, 1999], [Marschner]

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Isotropic BRDF


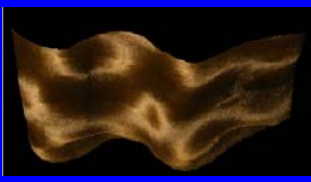
$$f(\theta_o, \varphi_o, \theta_i, \varphi_i) = f(\theta_o, \theta_i, \varphi_i - \varphi_o)$$

From Hertzmann & Seitz, CVPR'03

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Anisotropic BRDF

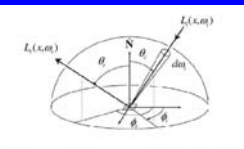



From Hertzmann & Seitz, CVPR'03

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Lambertian (Diffuse) Surface

- BRDF is a constant called the albedo a
- Emitted radiance is NOT a function of outgoing direction – i.e. constant in all directions.
- For lighting coming in single direction, emitted radiance is proportional to cosine of the angle between normal and light direction



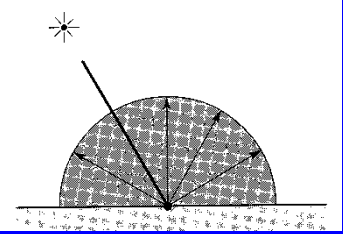
$$L_r(x, \omega_r) = \int_{\omega_i} f_r(x, \omega_i \rightarrow \omega_r) L_i(x, \omega_i) \cos \theta_i d\omega_i$$

$$L_r = aN \cdot \omega_l$$

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Lambertian reflection

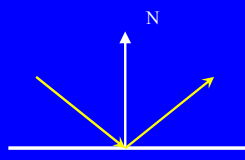
- Lambertian
- Matte
- Diffuse



Light is radiated equally in all directions

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Specular Reflection: Smooth Surface



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Rough Specular Surface

Average surface normal

Symmetric V-shaped grooves - 'microfacets'

Phong Lobe

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Phong Model

Mirror

Diffuse

S

CS348B Lecture 10
Pat Hanrahan, Spring 2002

Non-Lambertian reflectance

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Surface Reflectance Models

Common Models

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Arbitrary Reflectance

- Non-parametric model
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General BRDF: e.g. Velvet

Portrait of Sir Thomas More, Hans Holbein the Younger, 1527

[After Koenderink et al, 1998]

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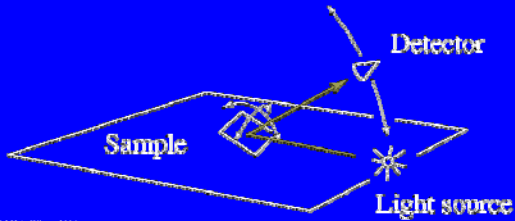
Gonioreflectometers

- For an isotropic BRDF
- Three degrees of freedom spread among light source, detector, and/or sample

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Gonioreflectometers

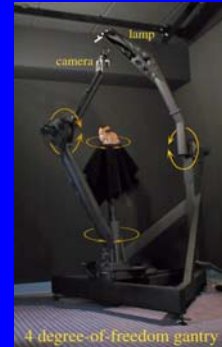
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Gonioreflectometers

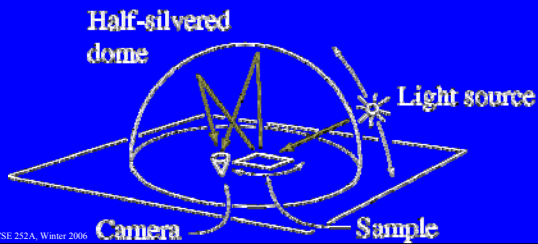
- Can add fourth degree of freedom to measure anisotropic BRDFs



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Ward's BRDF Measurement Setup

- Collect reflected light with hemispherical (should be ellipsoidal) mirror [SIGGRAPH 92]



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Ward's BRDF Measurement Setup

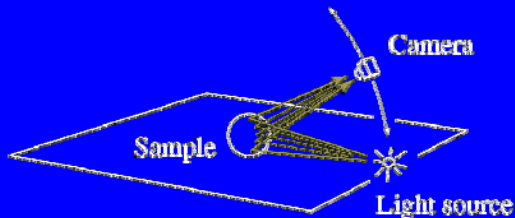
- Result: each image captures light at all exitant angles



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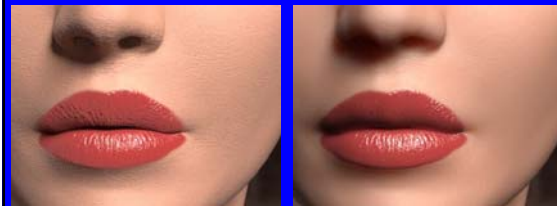
Marschner's Image-Based BRDF Measurement

- For uniform BRDF, capture 2-D slice corresponding to variations in normals



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BRDF Not Always Appropriate



BRDF

BSSRDF

<http://graphics.stanford.edu/people/panajis/>
(Jensen, Marschner, Levoy, Hanrahan)

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Light sources and shading

- How bright (or what colour) are objects?
- One more definition: Exitance of a source is
 - the internally generated power radiated per unit area on the radiating surface
- Similar to irradiance