

Image Formation and Cameras

Computer Vision I
CSE 252A
Lecture 3

CS252A, Winter 2006

Computer Vision I

Announcements

- Assignment 0: “Getting Started with Matlab” is posted to web page, due Thursday
- Read Chapters 1 & 2 of Forsyth & Ponce
- Add yourself to mailinglist

CS252A, Winter 2006

Computer Vision I

Image Formation: Outline

- Factors in producing images
- Projection
- Perspective/Orthographic Projection
- Vanishing points
- Projective Geometry
- Rigid Transformation and $SO(3)$
- Lenses
- Sensors
- Quantization/Resolution
- Illumination
- Reflectance and Radiometry

CS252A, Winter 2006

Computer Vision I

Earliest Surviving Photograph



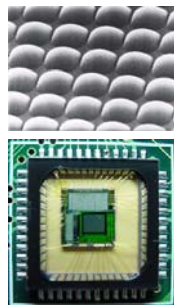
- First photograph on record, “la table service” by Nicéphore Niépce in 1822.
- Note: First photograph by Niépce was in 1816.

CS252A, Winter 2006

Computer Vision I

How Cameras Produce Images

- Basic process:
 - photons hit a detector
 - the detector becomes charged
 - the charge is read out as brightness
- Sensor types:
 - CCD (charge-coupled device)
 - high sensitivity
 - high power
 - cannot be individually addressed
 - blooming
 - CMOS
 - simple to fabricate (cheap)
 - lower sensitivity, lower power
 - can be individually addressed



CS252A, Winter 2006

Computer Vision I

Images are two-dimensional patterns of brightness values.

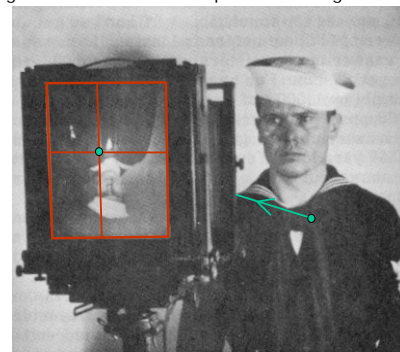


Figure from US Navy Manual of Basic Optics and Optical Instruments, prepared by Bureau of Naval Personnel, Reprinted by US Govt. Printing Office, June 1969.

CS252A, Winter 2006

They are formed by the projection of 3D objects. Computer Vision I

Effect of Lighting: Monet



CS252A, Winter 2006

Computer Vision I

Change of Viewpoint: Monet



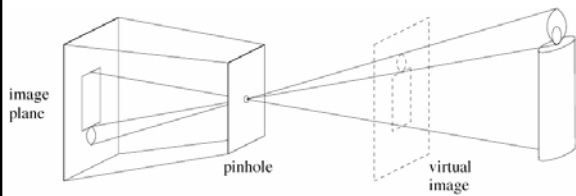
Haystack at Chailly at sunrise (1865)

CS252A, Winter 2006

Computer Vision I

Pinhole Camera: Perspective projection

- Abstract camera model - box with a small hole in it

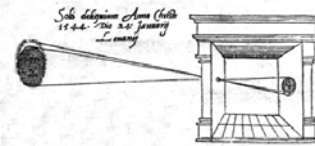


CS252A, Winter 2006 Forsyth&Ponce

Computer Vision I

Camera Obscura

illum in tabula per radios Solis, quibus in caelo contin-
gitur: hoc effectus in caelo superior pars deliquit pariatur, in
radiis apparebit inferior defecere, ut ratio exigit optica.



Sic nos exasit Anno .1544. Louani eclipsim Solis
obseruauimus, innumeratq; defecere paulo plus q; dex.

"When images of illuminated objects ... penetrate through a small hole into a very dark room ... you will see [on the opposite wall] these objects in their proper form and color, reduced in size ... in a reversed position, owing to the intersection of the rays".

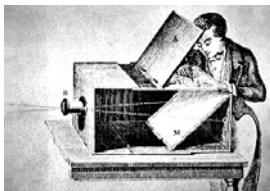
Da Vinci

http://www.acmi.net.au/AIC/CAMERA_OBSCURA.html (Russell Naughton)

CS252A, Winter 2006

Computer Vision I

Camera Obscura



- Used to observe eclipses (e.g., Bacon, 1214-1294)
- By artists (e.g., Vermeer).

CS252A, Winter 2006

Computer Vision I

Camera Obscura



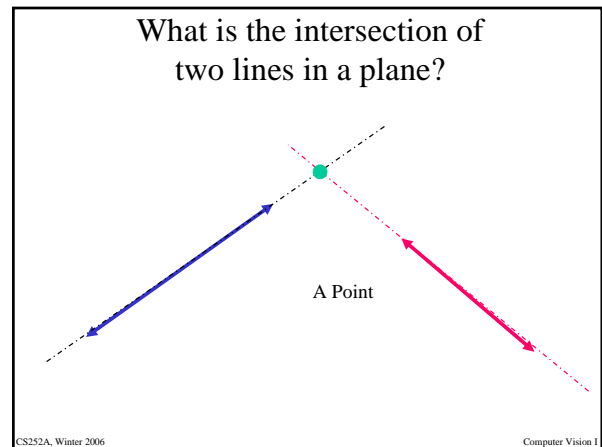
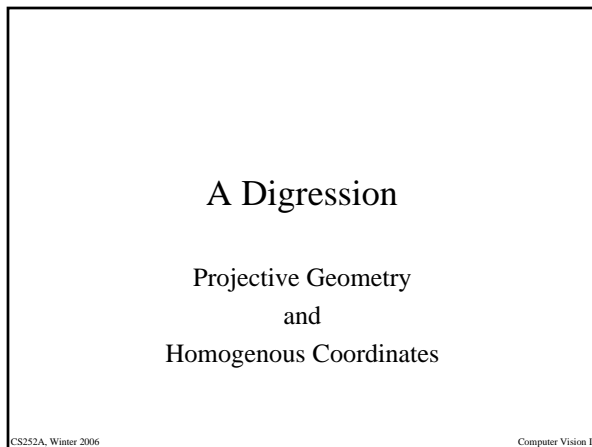
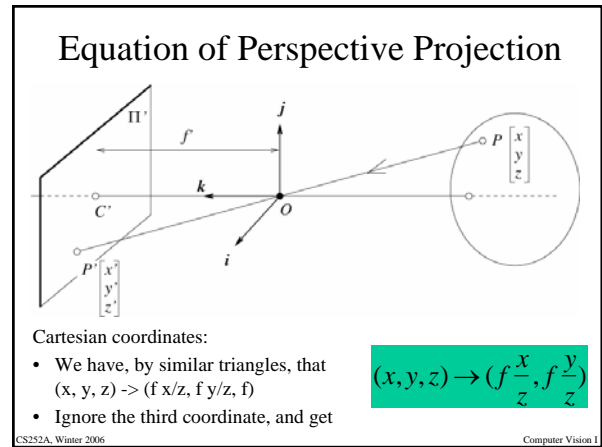
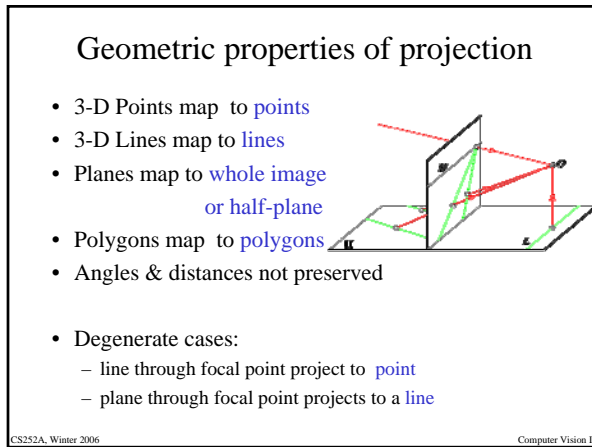
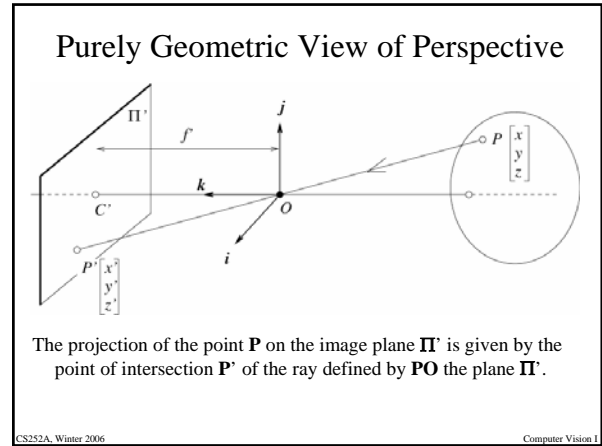
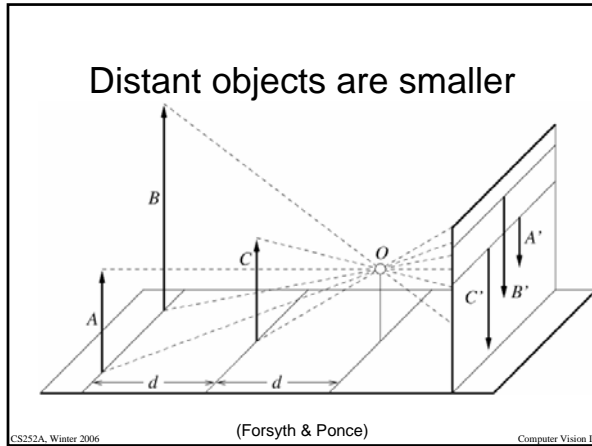
Jetty at Margate England, 1898.



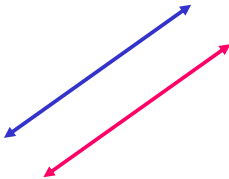
<http://brightbytes.com/cosite/collection2.html> (Jack and Beverly Wilgus)

CS252A, Winter 2006

Computer Vision I



Do two lines in the plane always intersect at a point?

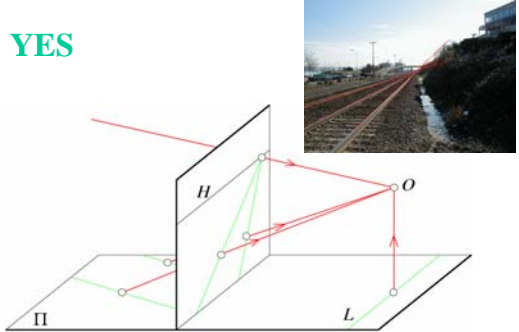


No, Parallel lines don't meet at a point.

CS252A, Winter 2006 Computer Vision I

Can the perspective image of two parallel lines meet at a point?

YES



CS252A, Winter 2006 Computer Vision I

Projective geometry provides an elegant means for handling these different situations in a unified way and **homogenous coordinates** are a way to represent entities (points & lines) in projective spaces.

CS252A, Winter 2006 Computer Vision I

Projective Geometry

- Axioms of Projective Plane
 1. Every two distinct points define a line
 2. Every two distinct lines define a point (intersect at a point)
 3. There exists three points, A,B,C such that C does not lie on the line defined by A and B.
- Different than Euclidean (affine) geometry
- Projective plane is "bigger" than affine plane – includes "line at infinity"

$\text{Projective Plane} = \text{Affine Plane} + \text{Line at Infinity}$

CS252A, Winter 2006 Computer Vision I

Homogenous coordinates
A way to represent points in a projective space

1. Use three numbers to represent a point on a projective plane
2. Add an extra coordinate
e.g., $(x,y) \rightarrow (x,y,1)$
2. Impose equivalence relation
 $(x,y,z) \approx \lambda \cdot (x,y,z)$
such that $(\lambda \neq 0)$
i.e., $(x,y,1) \approx (\lambda x, \lambda y, \lambda)$
3. Point at infinity – zero for last coordinate
e.g., $(x,y,0)$

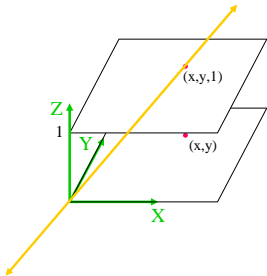
- Why do this?
 - Possible to represent points "at infinity"
 - Where parallel lines intersect
 - Where parallel planes intersect
 - Possible to write the action of a perspective camera as a matrix

CS252A, Winter 2006 Computer Vision I

Homogenous coordinates
A way to represent points in a projective space

1. Use three numbers to represent a point on a projective plane
2. Add an extra coordinate
e.g., $(x,y) \rightarrow (x,y,1)$

Impose equivalence relation
 $(x,y,z) \approx \lambda \cdot (x,y,z)$
such that $(\lambda \neq 0)$
i.e., $(x,y,1) \approx (\lambda x, \lambda y, \lambda)$



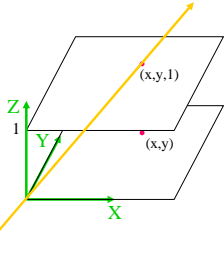
CS252A, Winter 2006 Computer Vision I

CONVERSION

Euclidean -> Homogenous -> Euclidean

In 2-D

- Euclidean -> Homogenous:
 $(x, y) \rightarrow k(x, y, 1)$
- Homogenous -> Euclidean:
 $(x, y, z) \rightarrow (x/z, y/z)$



In 3-D

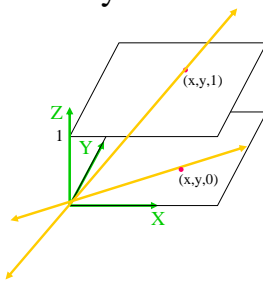
- Euclidean -> Homogenous:
 $(x, y, z) \rightarrow k(x, y, z, 1)$
- Homogenous -> Euclidean:
 $(x, y, z, w) \rightarrow (x/w, y/w, z/w)$

CS252A, Winter 2006 Computer Vision I

Points at infinity

Point at infinity – zero for last coordinate $(x,y,0)$ and equivalence relation $(x,y,0) \approx \lambda \cdot (x,y,0)$

No corresponding Euclidean point

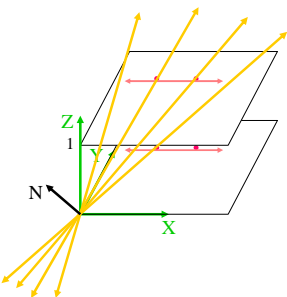


Projective Plane = Affine Plane + Line at Infinity

CS252A, Winter 2006 Computer Vision I

Lines in Projective space

1. Line in Euclidean plane
2. Plane in homogenous coordinates
3. Plane is represented by its normal \mathbf{N}
4. Equation for plane is $\mathbf{N} \cdot (x,y,z) = 0$
 $\mathbf{M} \cdot (x,y,z) = 0$
Where $\mathbf{M} = \lambda \mathbf{N}$



CS252A, Winter 2006 Computer Vision I

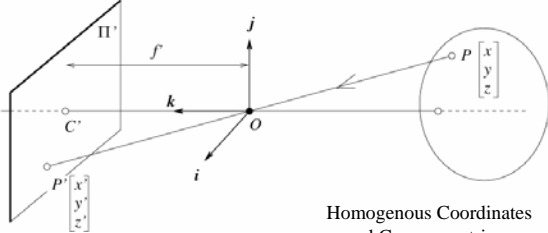
Projective transformation

- 3 x 3 linear transformation of homogenous coordinates
- Points map to points,
- lines map to lines

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

CS252A, Winter 2006 Computer Vision I

The equation of projection



Homogenous Coordinates and Camera matrix

Cartesian coordinates:

$$(x, y, z) \rightarrow \left(f \frac{x}{z}, f \frac{y}{z} \right)$$

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ T \end{bmatrix}$$

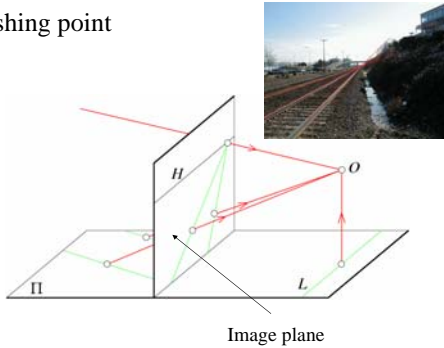
CS252A, Winter 2006 Computer Vision I

End of the Digression

CS252A, Winter 2006 Computer Vision I

Parallel lines meet in the image

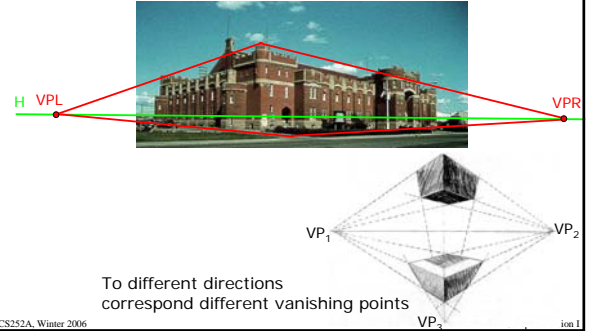
- vanishing point



CS252A, Winter 2006

Computer Vision I

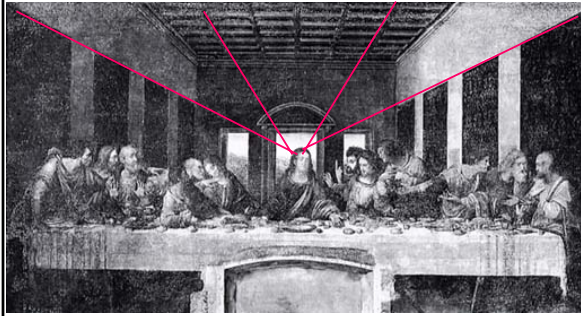
Vanishing points



CS252A, Winter 2006

ion I

Vanishing Points



CS252A, Winter 2006

Computer Vision I

Vanishing Point

- In the projective plane, parallel lines meet at a point at infinity.
- The vanishing point is the perspective projection of that point at infinity, resulting from multiplication by the camera matrix.

CS252A, Winter 2006

Computer Vision I

Affine Camera Model

- Take Perspective projection equation, and perform Taylor Series Expansion about (some point (x_0, y_0, z_0)).
- Drop terms of higher order than linear.
- Resulting expression is the affine camera model

CS252A, Winter 2006

Computer Vision I

Perspective

$$\begin{bmatrix} u \\ v \\ z \end{bmatrix} = \frac{f}{z} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Assume that $f=1$, and perform a Taylor series expansion about (x_0, y_0, z_0)

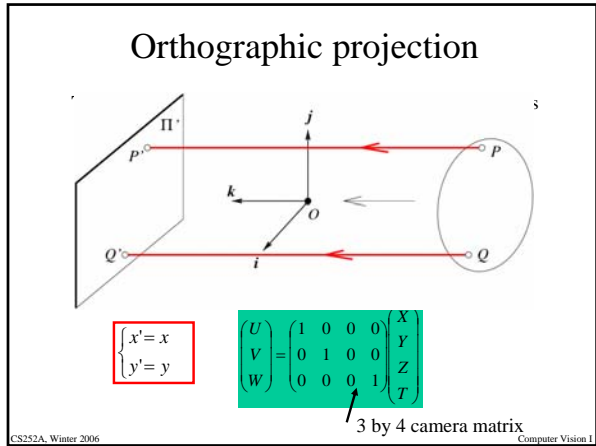
$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{1}{z_0} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \frac{1}{z_0} \begin{bmatrix} 1 \\ 0 \end{bmatrix} (x - x_0) + \frac{1}{z_0} \begin{bmatrix} 0 \\ 1 \end{bmatrix} (y - y_0) - \frac{1}{z_0^2} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} (z - z_0) + \frac{1}{2} \frac{2}{z_0^3} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} (z - z_0)^2 + \dots$$

Dropping higher order terms and regrouping.

$$\begin{bmatrix} u \\ v \end{bmatrix} \approx \frac{1}{z_0} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} 1/z_0 & 0 & -x_0/z_0^2 \\ 0 & 1/z_0 & -y_0/z_0^2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{A}\mathbf{p} + \mathbf{b}$$

CS252A, Winter 2006

Computer Vision I



- ### Properties of Affine or Orthographic camera models
- Parallel lines in the world, project to parallel lines.
 - Ratios of distances are preserved
 - Ratios of angles are preserved.
- CS252A, Winter 2006 Computer Vision I