Topological Sorting

Given a directed network (DAG) Can we index the nodes such that all directed arcs satisfy $(i, j)$ if $i < j$. 
Example
The Floyd-Warshall algorithm finds all shortest paths without negative cycles.

\[ p_{i,k} = k \]

\[
 p_{i,k} = \begin{cases} 
 p_{i,j} & \text{if } d_{i,k} > d_{i,j} + d_{j,k} \\
 \text{same} & \text{if } d_{i,k} \leq d_{i,j} + d_{j,k} 
\end{cases}
\]
Floyd-Warshall algorithms creates basic arcs updated from $j = 1, \ldots, n$.

Problem: Network is too large

Problem: $\exists$ negative cycles
Power Failure?

What happens if power is shut down?

Power shuts down...

\[ j = 1, \text{ all } i, k \neq j \]
\[ j = 2, \text{ all } i, k \neq j \]
\[ j = 3, \text{ all } i, k \neq j \]
After $j = 3$, all $i, k \neq j$
After $j = 1, 2, 3, 4$, $i, k \neq j$?
Computer can contain $|A| \cup |X|$ or $|X| \cup |B|$
but not $|A| \cup |X| \cup |B|$
Triple Operation on $A \cup X, B \cup X, A \cup X$