1 Jan 10, 2006

1.1 Symbols

∀ for all

∃ there exists

⇒ implies / only if / is sufficient for

⇔ if and only if / is necessary and sufficient for

∈ is an element of

1.2 BFS

Let 1, 2, \ldots, j, \ldots, k be the assigned labels and \( V_j \) has the \textit{smallest} label and unlabeled neighbors. Label the neighbor \( k + 1 \).

1.3 DFS

Let 1, 2, \ldots, j, \ldots, k be the assigned labels and \( V_j \) has the \textit{largest} label and unlabeled neighbors. Label the neighbor \( k + 1 \).

1.4 Graphs

Assuming \( d_{i,j} > 0 \):

- \(|\text{path}| \geq |\text{subpath}|\)
- Subpath of shortest path must itself be shortest
- Any shortest path contains at most \( n - 1 \) arcs
- \( P_1 \leq P_2 \leq P_3 \leq \cdots \leq P_{n-1} \)

1.5 Shortest Path (Dijkstra)

0. Vertes \( V_0 \) with \( l_0^* = 0 \)

- \( V_i \) get temp labels
- \( l_i = d_{0,i} \) (direct arc)
- \( l_i = \infty \) (no direct arc)

1. Pick \( l_k = \min_i l_i \)

2. \( l_i \leftarrow \min[l_i, l_k^* + d_{k,i}] \)
2 Jan 12, 2006

2.1 Min Spanning Tree (Prim)

0. Vertes $V_0$ with $l_0 = 0$
   $V_i$ get temp labels
   $l_i = d_{0,i}$ (direct arc)
   $l_i = \infty$ (no direct arc)

1. Pick $l_k = \min l_i$
   $l_k \leftarrow l^*_k$

2. $l_i \leftarrow \min[l_i, d_{k,i}]$

2.2 Min Spanning Tree Compared to Shortest Path

0. Same
1. Same
2. $l_i \leftarrow \min[l_i, d_{k,i}]$

In general, $l_i \leftarrow \min[l_i, \alpha l^*_k + \beta d_{k,i}]$

2.3 Other Optimal Paths

$L(e_1, e_2, e_3, \ldots, e_n) = L(L(e_1, e_2, e_3, \ldots, e_{n-1}), e_n)$

$L(e_1, e_2, e_3, \ldots, e_n, e_{n+1}) \geq L(e_1, e_2, e_3, \ldots, e_n)$

Then a path may be found where $L$ is minimized

2.4 Shortest Path Example

![Diagram]
3  Shortest Path From 0 to All

4  Minimum Spanning Tree

5  Minimum Clock Tree