Claim 1. Let $F \in 2\text{-SAT}$ and let $G$ be the digraph where $V(G)$ is the literals of $F$ and there is a directed edge from $a$ to $b$ iff $\{\overline{a}, b\} \in F$. Then $F$ is satisfiable iff $G$ has no cycle involving a literal and its negation.

Proof. ($\Rightarrow$) $\overline{a} \lor b$ is equivalent to $a \rightarrow b$. Suppose there is a path from $a$ to $\overline{a}$ to $a$ in $G$. Then by transitivity, the satisfiability of $F$ would imply that $a \leftrightarrow \overline{a}$, a contradiction.

($\Leftarrow$) We use induction on the number of variables. If $F$ has no variables, then $F$ is satisfiable, and we are done. Otherwise let $a$ be some literal in $G$ from which $\overline{a}$ cannot be reached.

Suppose indirectly that there is a path from $a$ to both some variable $b$ and its negation $\overline{b}$. Then by contraposition, there is also a path from $\overline{b}$ to $\overline{a}$, and by transitivity, there is a path from $a$ to $\overline{a}$, a contradiction.

Set all of the literals reachable from $a$ in $G$ to true. This is well-defined since no literal and its negation can both be reached from $a$. Furthermore, all clauses involving the literals just set are satisfied, and the graph $G'$ on the remaining formula also fails to have a cycle involving a literal and its negation. By the inductive hypothesis, the remaining formula is satisfiable, and we are done.

Claim 2. $2\text{-SAT} \in NL$.

Proof. Since NL = coNL, it is sufficient to show that $2\text{-SAT} \in NL$. We will demonstrate a log-space reduction from $2\text{-SAT}$ to PATH $\in NL$. So let $F \in 2\text{-CNF}$ have variable set $X$ and literal set $L$. Let $G$ be the digraph defined in claim 1.

For each $l \in L$, create a new copy $G_l$ of $G$:

$V(G_l) = V(G) \times \{l\}$
$E(G_l) = \{((a, l), (b, l)) \mid (a, b) \in E(G)\}$

and let $s, t$ be 2 nodes not in $V' = \bigcup_{l \in L} V(G_l)$. Define digraph $H$ by $V(H) = V' \cup \{s, t\}$ and

$E(H) = \bigcup_{l \in L} E(G_l)$
$\cup \{(s, (x, x)) \mid x \in X\}$
$\cup \{((\overline{x}, x), (\overline{x}, \overline{x})) \mid x \in X\}$
$\cup \{((x, \overline{x}), t) \mid x \in X\}$. 

The idea is that we can arrange the $G_t$ into 2 rows: the 1st row for those $G_x$ where $x \in X$ and the 2nd row for those $G_{\overline{x}}$ where $x \in X$. $s$ is above the 1st row and $t$ is below the 2nd row. For each $x \in X$, there is an edge from $s$ to the copy of $x$ in $G_x$, an edge from the copy of $\overline{x}$ in $G_x$ to the copy of $\overline{x}$ in $G_{\overline{x}}$, and an edge from the copy of $x$ in $G_{\overline{x}}$ to $t$.

We now show that there is an $x \in X$ and a path from $x$ to $\overline{x}$ to $x$ in $G$ iff there is a path from $s$ to $t$ in $H$.

$(\Rightarrow)$ Let $p$ be a path in $G$ from $x$ to $\overline{x}$ to $x$. Then the corresponding path in $H$ goes from $s$ to the copy of $x$ in $G_x$, follows the isomorphic copy of $p$ until reaching the copy of $\overline{x}$, then takes the edge from the copy of $\overline{x}$ in $G_x$ to the copy of $\overline{x}$ in $G_{\overline{x}}$, then continues to follow the isomorphic copy of $p$ until reaching the copy of $x$, and finally takes the edge from the copy of $x$ in $G_{\overline{x}}$ to $t$.

$(\Leftarrow)$ If $p$ is a path from $s$ to $t$ in $H$, then it must first enter $G_x$ for some $x \in X$. By stripping off $s, t$ from $p$ and the edge from $G_x$ to $G_{\overline{x}}$ and removing the 2nd component information in the nodes of $p$, we obtain a path in $G$ from $x$ to $\overline{x}$ to $x$.

So, by claim 1, $F \leftrightarrow H$ is a reduction from 2-SAT to PATH. The following algorithm shows that $H$ can be computed in log space:

1. for each $l \in L$ and $\{l_1, l_2\} \in F$
2. output $((l_1, l), (l_2, l), (l, l))$
3. for each $x \in X$
4. output $(s, (x, x)), ((\overline{x}, x), (x, \overline{x})), ((x, \overline{x}), t)$

Claim 3. 2-SAT $\in$ NL-hard.

Proof. We show a log-space reduction from PATH to 2-SAT which essentially encodes the phrase “there is a cut separating $s$ from $t$”. Let $G = (V, E)$ be a digraph and $s, t \in V$. For each $(x, y) \in E$, add the clause $\{\overline{x}, \overline{y}\}$ to $F$. Also add the unit clauses $\{s\}, \{\overline{t}\}$ to $F$. Clearly $F$ can be computed in log space. It remains to show that $(G, s, t) \in$ PATH iff $F \in$ 2-SAT.

$(\Rightarrow)$ If there is no path from $s$ to $t$, then there is a cut separating $s$ from $t$. Set the variables in the cut to true and the others to false to satisfy $F$.

$(\Leftarrow)$ If $F$ is satisfied by some assignment $a$, then the true variables in $a$ form a cut separating $s$ from $t$ and so there is no path from $s$ to $t$. 

\[\square\]