Let $L$ be the following problem: given digraph $G = (V, E)$ and $S \subseteq V^2$, decide whether there is a path $p$ from $s$ to $t$ s.t. $\forall (u, v) \in S$, $p$ uses at most one of $u, v$.

**Claim 1.** $L \in \text{NP-complete}$. 

**Proof.** $L \in \text{NP}$ since a witness is such a path $p$ and if there is any such path then there is one of length at most $|V|$. A path of polynomial size can easily be verified in polynomial time.

To show that $L$ is NP-hard, we reduce from 3-SAT: Let $F = \{C_1, \ldots , C_m\} \in 3\text{-CNF}$ be a set of $m$ 3-clauses, where a 3-clause is a set of $\leq 3$ literals. Let $V' = \{(l, C) \mid l \in C \in F\}$ and $V = V' \cup \{s, t\}$ where $s, t \notin V', s \neq t$. So $V$ is the set of literal occurrences of $F$, together with 2 new nodes $s, t$. Let 

$$E = \{((k, C_i), (l, C_{i+1})) \in V^2 \mid i \in \{1, \ldots , m-1\}\}$$

$$\cup\{(s, (l, C_1)) \in \{s\} \times V\} \cup \{((l, C_m), t) \in V \times \{t\}\}.$$ 

So there is an edge from each literal occurrence in $C_i$ to each literal occurrence in $C_{i+1}$, and edges from $s$ to the literal occurrences of $C_1$, and edges from the literal occurrences of $C_m$ to $t$. Let 

$$S = \{((k, C), (l, D)) \in V^2 \mid k, l \text{ are the negations of each other}\}.$$ 

We claim that $F \in \text{SAT}$ iff $(G, s, t, S) \in L$. If $F \in \text{SAT}$ with satisfying assignment $a$, then define $p$ to start at $s$, select a literal occurrence true at $a$ from each successive clause, and then end at $t$. $p$ does not violate any of the constraints in $S$ since $a$ is a consistent assignment.

Conversely, if $(G, s, t, S) \in L$, then let $p$ be a satisfying path from $s$ to $t$. Define variable assignment $a$ as follows: if $p$ passes through variable $x$, then assign $x$ true, if $p$ passes through $\neg x$, then assign $x$ false, otherwise assign $x$ arbitrarily. $a$ is well-defined since the constraints in $S$ preclude $p$ from passing through both $x$ and $\neg x$. Since for each clause $C$, $p$ passes through at least 1 literal of $C$, $a$ must satisfy $C$, and hence $F$. So $F \in \text{SAT}$.  

\[\square\]