A function $f$ of one variable is *strictly decreasing* if and only if for all real numbers $x$ and $y$ with $x < y$, $f(x) > f(y)$. Show that $f(x) = -x^3$ is strictly increasing. (Hint: $\forall x, y \in \mathbb{R}$ where either $x \neq 0$ or $y \neq 0$, $x^2 + xy + y^2 > 0$.)

Proof Without the loss generality, assume that $x < y$. (Note that $y - x > 0$ is positive since $x \neq y$.) From our assumption, we have $x^2 + xy + y^2 > 0$ as positive. Then, $(y - x)(x^2 + xy + y^2) > 0$ is positive since the product of two positive number is positive. Thus we have

\[
\begin{align*}
0 &< (y - x)(x^2 + xy + y^2) \\
0 &< y^3 - x^3 \\
x^3 &< y^3 \\
-x^3 &> -y^3 \\
f(x) &> f(y)
\end{align*}
\]

Therefore $f(x)$ is strictly decreasing. $\square$