1 Make a truth table for 

\[(p \land q) \lor \sim (p \lor \sim q)\].

\[
\begin{array}{c|c|c|c|c}
 p & q & (p \land q) & (p \lor \sim q) & \sim (p \lor \sim q) \\
 \hline
 0 & 0 & 0 & 1 & 0 \\
 0 & 1 & 0 & 0 & 1 \\
 1 & 0 & 0 & 1 & 0 \\
 1 & 1 & 1 & 0 & 1 \\
\end{array}
\]

Note that this function is simply \(q\).

2 Find a circuit with at most four logic gates (NOT, OR, AND), each of which is allowed to have at most two inputs, that equal the Boolean function defined by the following truth table:

\[
\begin{array}{c|c|c|c}
 p & q & r & S \\
 \hline
 0 & 0 & 0 & 1 \\
 0 & 0 & 1 & 0 \\
 0 & 1 & 0 & 1 \\
 0 & 1 & 1 & 1 \\
 1 & 0 & 0 & 0 \\
 1 & 0 & 1 & 0 \\
 1 & 1 & 0 & 0 \\
 1 & 1 & 1 & 0 \\
\end{array}
\]

This function reduces to \(\sim p \land (q \lor \sim r)\). To see this, we start by writing down the CNF formula and then simplifying using Boolean Algebra:

\[
(\sim p \land q \land \sim r) \lor (\sim p \land q \land \sim r) \lor (\sim p \land q \land r) = \\
= \sim p \land [(\sim q \land \sim r) \lor (q \land \sim r)] \\
= \sim p \land [(\sim r \land (\sim q \lor r)) \lor (q \land r)] \\
= \sim p \land [\sim r \lor (q \land r)] \\
= \sim p \land [(\sim r \lor q) \land (\sim r \lor r)] \\
= \sim p \land (\sim r \lor q)
\]