IS: Solutions to 1.7, 1.16, 1.4, 2.1, 2.2, 2.3, and the review questions (1, 2, 4, 7) are found in the back of your textbook.

IS 1.4 (using the perturbation method): Let

\[ S_n = \sum_{k=1}^{n} k^3 \]

Then \( S_{n+1} \) can be written as

\[
S_n + (n + 1)^3 = 1 + \sum_{k=1}^{n+1} k^3 \\
= 1 + \sum_{k=1}^{n} (k + 1)^3 \\
= 1 + \sum_{k=1}^{n} (k^3 + 3k^2 + 3k + 1) \\
= 1 + S_n + \sum_{k=1}^{n} 3k^2 + \sum_{k=1}^{n} 3k + \sum_{k=1}^{n} 1
\]

The \( S_n \)'s cancel out, leaving us with:

\[
3 \sum_{k=1}^{n} k^2 = (n + 1)^3 - 1 - 3 \sum_{k=1}^{n} k - n \\
= (n + 1)^3 - \frac{3(n)(n+1)}{2} - (n + 1) \\
= (n + 1)((n + 1)^2 - 3n/2 - 1) \\
= (n + 1)n(n + 1/2)
\]
Then we have
\[
\sum_{k=1}^{n} k^2 = \frac{(n)(n + 1)(2n + 1)}{6}
\]

Pumpkins: The problem with the proof lies in using the incorrect base case. If we could prove a base case for sets of size 2, the proof would hold. That is, if all pumpkins are pair-wise equal, then all pumpkins are equal.

However, if we apply the same argument we used to get from sets of size \( n - 1 \) to \( n \) on sets of size 1, we find a problem. Since the intersection of two sets of size 1 is empty, we cannot use transitivity to get sets of size 2. The proof by induction depends on using sets of size 2 to get to sets of size 3, and so on. This is why the proof breaks down.

Predicates: The following predicates are true:
- \( P(2) \)
- \( P(n) \rightarrow P(n-1) \) for \( n \geq 1 \)
- \( (P(2) \text{ AND } P(n)) \rightarrow P(2n) \)

Prove that \( P(i) \) is true for all non-negative integers \( i \).

Proof by induction:
Base case (\( P(1) \) is true): \( P(2) \rightarrow P(1) \), since \( P(2) \) and \( P(n) \rightarrow P(n-1) \).
Base case (\( P(0) \) is true): \( P(1) \rightarrow P(0) \), since \( P(1) \) and \( P(n) \rightarrow P(n-1) \).

Inductive Hypothesis: \( P(n) \) is true.

We will show that \( P(n+1) \) is true.
- \( P(n) \) is true by the inductive hypothesis
- \( P(2n) \) is true, since \( (P(2) \text{ AND } P(n)) \rightarrow P(2n) \)
- \( P(2n - 1) \) is true, since \( P(2n) \rightarrow P(2n-1) \)
- By the same reasoning, \( P(2n), P(2n-1), \ldots, P(n+1), P(n), \ldots, P(1) \) are all true.
- Therefore \( P(n+1) \) is true.

We have shown, using a proof by induction, that \( P(i) \) is true for all non-negative integers \( i \).