1. Prove using the element method that, for all subsets, $P, Q, R$ of $U$, $Q - (P \cap R) = Q - (P \cap Q \cap R)$.

NOTE: To gain intuition, it might be helpful to try the proof using Venn diagrams or set algebra.

NOTE: When proving using the element method, we have that $LHS \subseteq RHS$ and $RHS \subseteq LHS$ in order to conclude $LHS = RHS$. (In our example the left-hand set (LHS) is $Q - (P \cap R)$ and the right-hand set (RHS) is $Q - (P \cap Q \cap R)$). Each subproof requires that we assume that $x$ is an element of the subset and show that $x$ is an element of the superset.

First we show that $Q - (P \cap R) \subseteq Q - (P \cap Q \cap R)$. Let $x \in Q - (P \cap R)$. Then $x \in Q$ and $x \notin P \cap R$. Then $x \notin (P \cap Q \cap R)$ since $P \cap Q \cap R$ is a subset of $P \cap R$. Then $x \in Q \cap (P \cap Q \cap R)^c$, or similarly by the definition of set difference, $x \in Q - (P \cap Q \cap R)$.

Next we show that $Q - (P \cap Q \cap R) \subseteq Q - (P \cap R)$. Let $x \in Q - (P \cap Q \cap R)$. Then $x \in Q$ and $x \notin P \cap Q \cap R$. Then we know that $x \notin (P \cap R)$ because we know $x$ is in $Q$ but not in either $P$ or $R$, (or not in both $P$ and $R$). Therefore $x$ cannot be in the intersection of $P$ and $R$. Then $x \in Q \cap (P \cap R)^c$, or similarly $x \in Q - (P \cap R)$.

Since $Q - (P \cap Q \cap R) \subseteq Q - (P \cap R)$ and $Q - (P \cap R) \subseteq Q - (P \cap Q \cap R)$, $Q - (P \cap R) = Q - (P \cap Q \cap R)$.