Quiz 2 Solutions — CSE 105, Winter 2006

Problem 1 [10 points]

Give a context-free grammar for the language $L \subseteq \{0, 1, \sharp\}^*$, where

$$L = \{x\sharp y \mid x \neq y^R,\ |x| = |y|\}.$$

For example, $00\sharp10, 101\sharp000 \in L$, but $00\sharp0, 11\sharp11 \notin L$.

For a string $x\sharp y$ to be in this language, x must differ from y^R in at least one spot. Once this condition has been satisfied, any subsequent symbols are irrelevant, as long as the length constraint is satisfied.

$$\begin{aligned} S &\to 0S0 \mid 1S1 \mid D \\ D &\to 0A1 \mid 1A0 \\ A &\to 0A0 \mid 1A1 \mid 0A1 \mid 1A0 \mid \sharp \end{aligned}$$

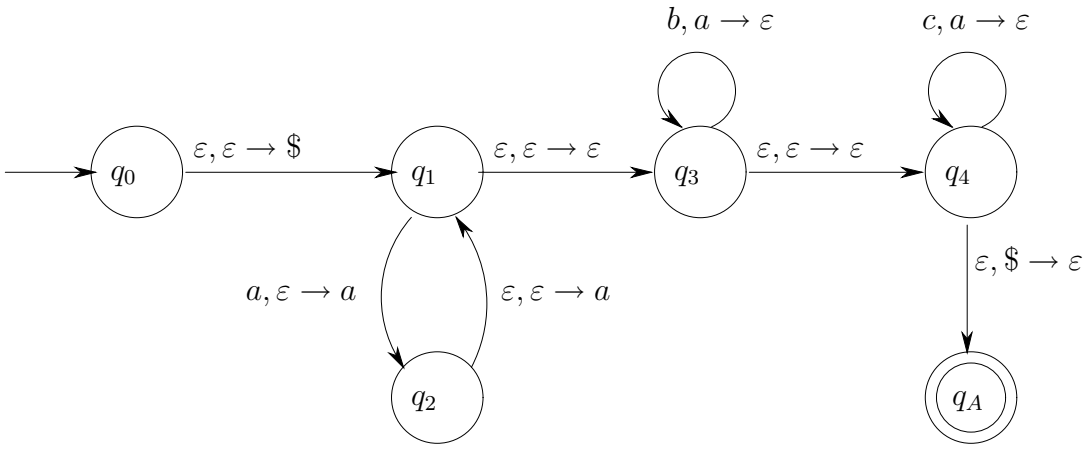
Problem 2 [10 points]

Give the state transition diagram for a pushdown automaton that recognizes the language

$$L = \{a^i b^j c^k \mid i, j, k \geq 0,\ k = 2i - j\},$$

e.g. $abc, abb, aabccc \in L$, but $aaabc, bcc \notin L$.

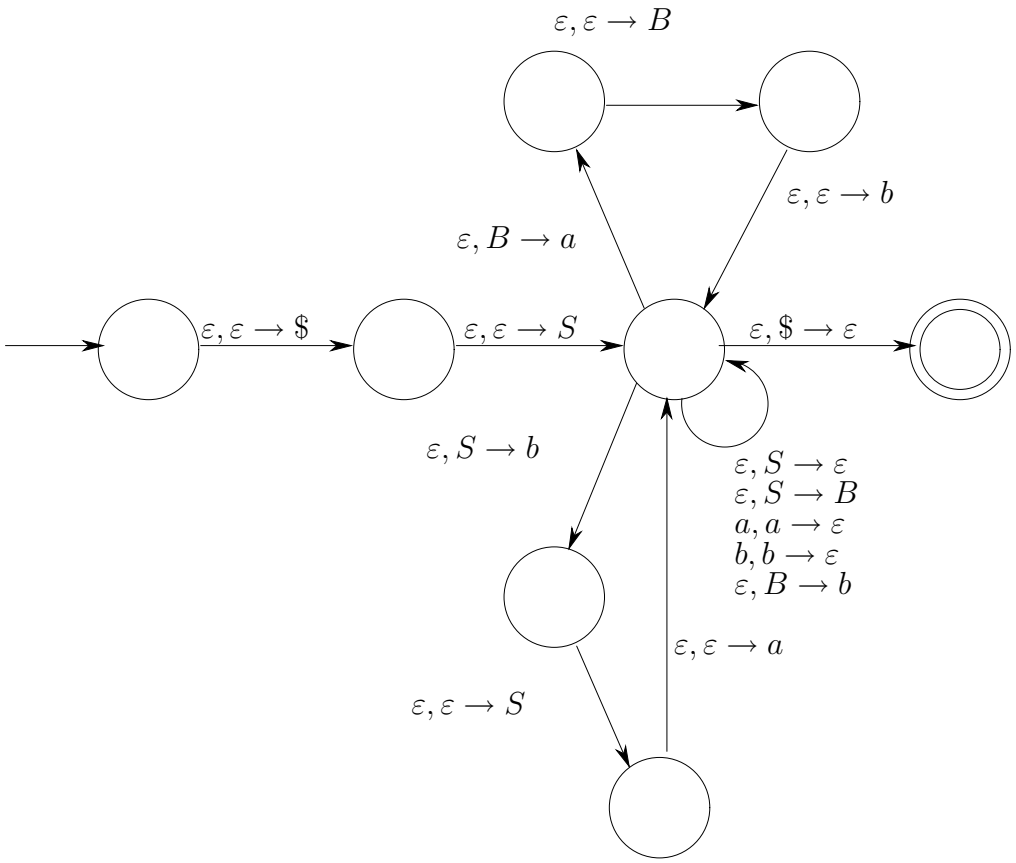
In this language, the number of bs plus the number of cs must equal twice the number of as. A PDA to recognize this language operates by first reading as and pushing two symbols onto the stack for every a read. It then transitions into a state to read bs, popping one symbol from the stack for each b read. Finally, it transitions into another state to read cs, and accepts when the stack is empty and all input has been read.



Problem 3 [6 points]

Transform the following CFG into an equivalent PDA by using the method discussed in class.

$$S \to aSb \mid B \mid \varepsilon$$
$$B \to bBa \mid b$$



Problem 4 [6 points]

Prove that the following context-free grammar is ambiguous.

$$S \to aSbT \mid T$$
$$T \to aT \mid bT \mid \varepsilon$$

Consider the string ab. There are two possible left-most derivations under this grammar.

- $S \Rightarrow aSbT \Rightarrow aTbT \Rightarrow a\varepsilon bT \Rightarrow ab\varepsilon = ab$
- $S \Rightarrow T \Rightarrow aT \Rightarrow abT \Rightarrow ab\varepsilon = ab$

Therefore, the grammar is ambiguous.

Problem 5 [8 points]

For each of the following languages, determine whether the language is regular, context-free, or neither. You do not need to prove your answers. If a language is regular, it is not necessary to mark "context-free" as well. Assume that each language is over the alphabet $\Sigma = \{0, 1, 2\}$.

	Regular	Context-Free	Neither
$\{1^n(0+1)^*1^n \mid n \geq 4\}$	✓		
$\{2^*1^m0^n \mid m, n \geq 0,\ m+n \text{ is odd}\}$	✓		
$\{x2y \mid x, y \in \{0,1\}^*, \text{ for all } i \geq 0,\ x_i \neq y_i\}$			✓
$\{0^m21^n0^* \mid m, n \geq 0, n \leq 2m\}$		✓	