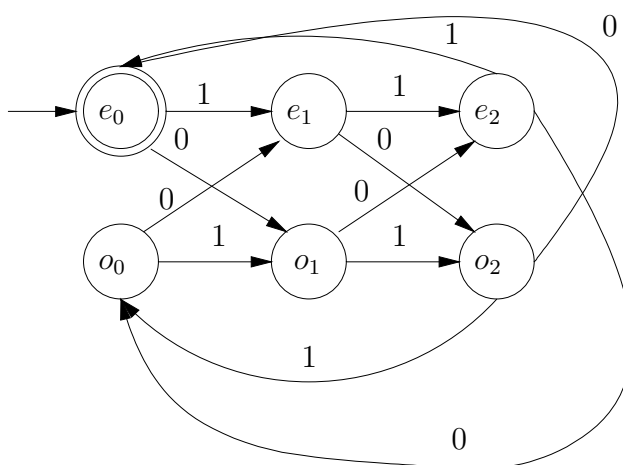


Quiz 1 Solutions — CSE 105, Winter 2006

Problem 1 [6 points]

Let $L \subseteq \{0, 1\}^*$ be the language of all strings w containing an even number of 0's where $|w|$ is a multiple of 3. (E.g. $011101 \in L$, but $1001 \notin L$.) Draw the state diagram of a DFA that accepts L .

We can view this language as the intersection of two languages: $L_3 = \{w \mid |w| \bmod 3 = 0\}$, and $L_{\text{even}} = \{w \mid w \text{ contains an even number of 0s}\}$. A DFA for L can be designed using the cartesian product construction.



States e_i represent strings of length $i \pmod{3}$ with even numbers of 0s, and o_i have odd numbers of zeros. Each symbol read increments $i \pmod{3}$, and if the symbol is a 0, the machine transitions from the set of even states to the set of odd states.

Problem 2 [4 points]

Give a regular expression for the language $L \subseteq \{a, b\}^*$ of all non-empty strings w that satisfy the following properties:

- if w begins with a , it must contain exactly three b s,
- if w begins with b , it cannot contain bb as a substring.

(E.g. $abbabaa$ and $babaaabab \in L$, but $ababaaa$ and $bbaab \notin L$.)

$$a(a^*ba^*ba^*ba^*) + b(a + ab)^*$$

Problem 3 [8 points]

Transform the following NFA into an equivalent DFA using the procedure studied in class:

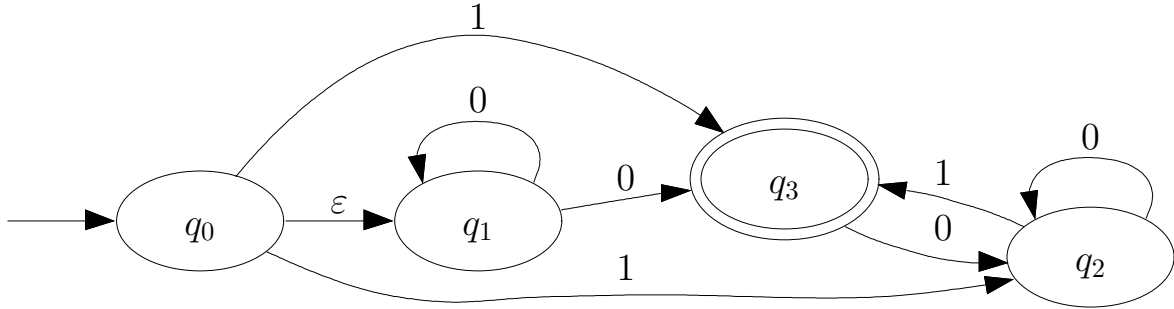
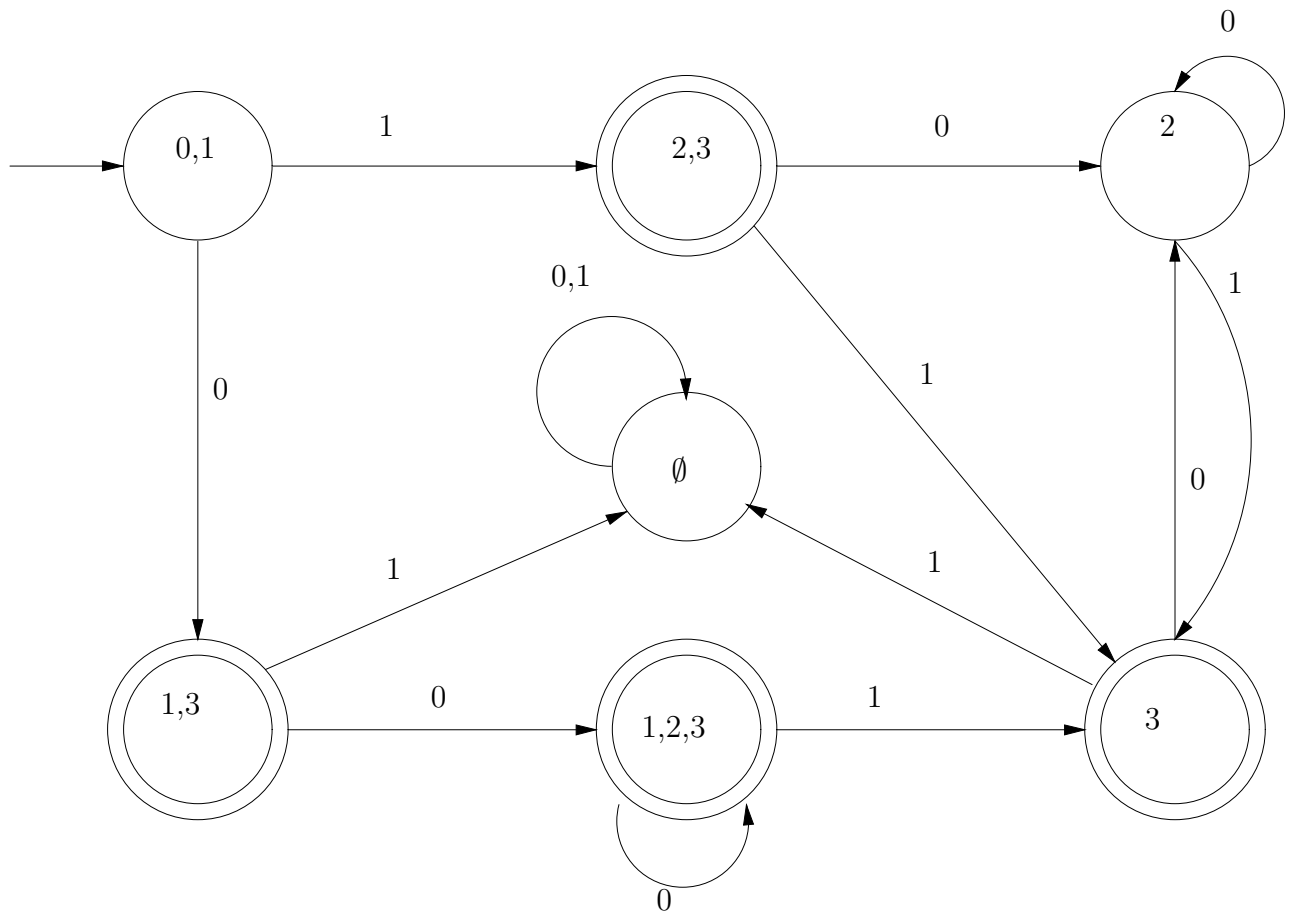


Figure 1: NFA

You only need to draw the part of the diagram consisting of the reachable states. Specifically, all you need to do is to complete the following diagram (which shows the reachable states) by marking the start state, accept states, and transitions.



Problem 4 [12 points]

Indicate whether the following statements are TRUE or FALSE by **circling** correct answers. Briefly justify your answers by writing 1 or 2 lines of explanation.

(a) $(01)^* = 0^*1^*$

FALSE: $(01)^*$ generates the string 0101, which cannot be generated by 0^*1^* .

(b) Any regular language can be recognized by an NFA with only one accept state.

TRUE: Let $N = (Q, \Sigma, \delta, q_0, F)$ be an NFA recognizing some language. We can construct $N' = (Q \cup q_F, \Sigma, \delta', q_0, \{q_F\})$ as follows:

$$\delta'(q, \varepsilon) = \begin{cases} q_F & \text{if } q \in F \\ \delta(q, \varepsilon) & \text{otherwise} \end{cases}$$

On non-empty transitions, $\delta' = \delta$.

(c) If $M = (Q, \Sigma, \delta, q_0, F)$ is a DFA and $q_0 \notin F$, then $\varepsilon \notin \mathcal{L}(M)$.

TRUE: If $q_0 \notin F$, then the only way to reach an accept state is to read an input symbol.

(d) If $M = (Q, \Sigma, \delta, q_0, F)$ is an NFA and $q_0 \notin F$, then $\varepsilon \notin \mathcal{L}(M)$.

FALSE: In an NFA, it is possible to transition from q_0 to an accept state on an empty string.

(e) Given an NFA with n states, any equivalent DFA must have at least $n + 1$ states.

FALSE: If an NFA N 's transition function δ is complete, deterministic, and has no ε -transitions, then converting N to a DFA will not require additional states.

(f) If L is a non-regular language, there exists some language A such that $L \cap A$ is regular.

TRUE: If $A = \emptyset$, then $L \cap A = \emptyset$, which is regular.

Problem 5 [10 points]

Prove that the language

$$L = \{x1^i y : i \geq 0, x, y \in \{0, 1\}^*, |x| \leq i, |y| \leq i\}$$

is not regular using the pumping lemma.

Let p be the pumping length, and $s = 0^p 1^p 0^p$. Clearly, $s \in L$. Now, assume that L is regular, so that the pumping lemma holds. By the pumping lemma, we can decompose s into substrings uvw , such that v is non-empty, and $|uv| \leq p$. Then $uv^i w \in L$ for any $i \geq 0$.

Because $|uv| \leq p$, v must contain only 0s. If we let $i = 2$ (pump up), then $uv^2 w = 0^{p+|v|} 1^p 0^p$. Since $p + |v| > p$, we know that $uv^2 w \notin L$, which violates the pumping lemma. Therefore, L cannot be regular.